

PC5215, Numerical Recipes with Applications

Lab 1, due ~~Friday, 8~~ Monday 12 September 2023

Submit in PDF at Canvas your codes with a report of the steps and analysis, as well as answers. Just the codes without running data or missing English description is not acceptable.

1. Use the Python version of the “Numerical Recipes” LU decomposition functions [ludcmp(), lubksb()] to solve the following 4×4 linear equations, and verify, by hand or using other software, such as Matlab, or a consistent check, that the answers are correct.

$$\begin{bmatrix} 1 & 3 & 3 & -5 \\ 2 & -4 & 7 & -1 \\ 7 & 1/2 & 3 & -6 \\ 9 & -2 & 3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \\ -10 \end{bmatrix}$$

2. Consider square grids of $L \times L$ resistor network shown below on the next page (for the case of $L=4$). Given that point A has voltage 0 volt and B voltage 1 volt, compute the total current flowing from B to A, in units of ampere, and the total resistance of the network. We assume that each link has a resistance $r = 1 \Omega$. Use the same linear solver as in problem 1 above. Report the results on $L = 1, 2, 4, 8, 16, 32, \dots$, in the power of two (as large as you can get), and the times needed (using Python time()). Then compare the answers and times with scipy linalg.solve() results as well. Also, check your computer answer against hand calculation for the case of $L=2$.

To set up the linear equations, you need (1) Ohm’s law $I_{ij} = (V_j - V_i)/r$ for each link, (2) conservation of current $\sum I_{ij} = 0$ at each node. And make linear equations for the voltage at each node. You need a good naming convention for the sites in order to construct the matrix A , and figure out what is b , such that $Ax=b$, where x is the voltage of each node. Pay attention to the difference between the internal node which has four neighbors vs those at the edge which has less number of neighbors. Be careful about the sign of the current. The program should work for general L as an input parameter.

