Tutorial No 6 and Answers

1. Starting with a waveform with a rich set of harmonics, we wish to synthesize a waveform whose harmonics between 500 Hz and 2000 Hz are absent or very weak. what kind of filter or filters would have to be used to remove or weaken all the frequencies 500 Hz and 2000 Hz?

   Answer: We could first use a *low pass filter* to remove all the frequencies above 500 Hz. Taking the original waveform again, we could use a *high pass filter* to remove all the frequencies below 2000 Hz. We could then add up both resultant waveforms, which would give us the frequencies below 500 Hz and the frequencies above 2000 Hz. A more efficient method would be use a *band reject filter* or *band stop filter* which could simultaneously reject all frequencies between 500 hz and 2000 Hz. A *band pass filter* would do the opposite i.e. reject all frequencies outside a certain band of frequencies.

2. When a waveform is quantized, the number of quantization levels depends on the bit length of the binary number to be used for the digitization. In general, a bit length of n gives 2^n levels. For example, a bit length of 2 bits will give 2^2 or 4 quantization levels, 4 bits will give 2^4 or 16 levels, 8 bits will give 2^8 or 256 levels, and 16 bits will give 2^{16} or 65,536 levels. How many quantization levels will we get for binary numbers with bit lengths of 3 bits, 5 bits and 6 bits? Which bit length preserves the waveform more accu-

rately?

Answer: A bit length of 3 bits will give 2^3 or 8 quantization levels, a bit length of 5 bits will give 2^5 or 32 quantization levels, while a bit length of of 6 bits will give 2^6 or 64 quantization levels. Since more quantization levels means that the waveform is better preserved, a 6-bit length is better than a 3-bit or 5-bit length for waveform preservation.

3. You are recording the NUS Choir digitally, and in the quantization process you use a bit length of 10 bits. What would be a fair approximation for the value of signal-to-noise (S/N) ratio due to quantization noise?

   Answer: Generally speaking, the signal-to-noise ratio due to quantization noise depends directly on the bit length of the samples. As a rule of thumb, we can say that each bit of the bit length will give about 6 dB of signal-to-noise ratio. Therefore we would expect a bit length of 10 bits to give a signal-to-noise ratio of about 10 times 6 i.e. 60 dB. (10 bits gives 2^{10} or 1,024 quantization levels, and a noise amplitude which is 1,023 times less than the signal. The ratio of the signal power to the noise power is $1,023^2$ or 1,046,529 to 1, giving a signal-to-noise ratio in terms of dB equal to 60.19 dB. You do not need to know this kind of calculation; all you need to know is that 1 bit will give about 6 dB of signal-to-noise ratio.)

4. A concert by the NUS Orchestra is being digitally broadcast over the Internet, and you wish to ensure that all frequencies up to at least 16 kHz are pre-

served in the recording. What should be the minimum sampling frequency i.e. the rate at which samples are produced, of the resulting digital signal? If the Internet broadcast can only transmit a maximum bit rate of 800,000 bits per second, what is the best signal-to-noise ratio possible in this broadcast? (Assume that the recording is in stereo sound with two audio channels.)

Answer: The Nyquist theorem or criterion says that to preserve a frequency of f Hz when it is being sampled, the sampling frequency must be at least $2f$ Hz. The sampling frequency to preserve 16 KHz should thus be at least 32,000 samples per second for each audio channel. The maximum bit rate is 800,000 bits per second, so since the broadcast is in stereo with two audio channels, each audio channel has a maximum bit rate of 400,000 bits per second (or 50,000 bytes per second since 8 bits equals 1 byte). We can then deduce that the maximum possible number of bits per sample is given by 400,000 bits per second divided by 32,000 samples per second i.e. 12.5 bits. However, the number of bits must be an integer so it has to be no more than 12 bits. A bit length of 13 bits would result in a bit rate of 13 bits times 32,000 samples per second, i.e. 416,000 bits per second which exceeds the maximum possible bit rate of 400,000 bits per second. A bit rate of 12 bits will give a best possible signal-to-noise ratio of 12 times 6 dB i.e. 72 dB.

5. In the MP3 system, we can compress the digital in-

formation coming from a normal audio CD so that it takes much fewer bits to store the music in an MP3 file. The most common bit rate in MP3 is 128 Kbits per second. If one of your favourite songs on a CD track takes up 30 Mbytes (30 million bytes), how many bytes would you expect it to take up on an MP3 file? Assume that the CD bit rate is 1,411,200 bits per second.

Answer: The ratio of the CD bit rate to the MP3 bit rate is 1,411,200 to 128,000. This gives a compression ratio of 11.025. The 30 Mbyte file would thus be compressed by this ratio to give 30,000,000 Mbytes divided by 11.025 i.e. 2,721,088 bytes or about 2.72 Mbytes.