Preface

This book on the quantum mechanics of *Simple Systems* grew out of a set of lecture notes for a third-year undergraduate course at the National University of Singapore (NUS). The reader is expected to have the minimal knowledge of a standard brief introduction to quantum mechanics with its typical emphasis on one-dimensional position wave functions.

Proceeding from there, Dirac's formalism of kets, bras, and all that is introduced immediately. In this natural language of the trade, the elementary situations of no force, constant force, and linear restoring force are then dealt with in considerable detail, with Schrödinger's and Heisenberg's equations of motion on equal footing. After treating orbital angular momentum and hydrogen-like atoms, there follows a final chapter on approximation methods, from the Hellmann–Feynman theorem to the WKB quantization rule. For the benefit of the learning student, intermediate steps are not skipped and dozens of exercises are incorporated into the text.

Two companion books on *Basic Matters* and *Perturbed Evolution* cover the material of the preceding and subsequent courses at NUS for second- and fourth-year students, respectively. The three books are, however, not strictly sequential but rather independent of each other and largely self-contained. In fact, there is quite some overlap and a considerable amount of repeated material. While the repetitions send a useful message to the self-studying reader about what is more important and what is less, one could do without them and teach most of *Basic Matters*, *Simple Systems*, and *Perturbed Evolution* in a coherent two-semester course on quantum mechanics.

All three books owe their existence to the outstanding teachers, colleagues, and students from whom I learned so much. I dedicate these lectures to them.

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I wish to thank my dear wife Ola for her continuing understanding and patience by which she is giving me the peace of mind that is the source of all achievements.

Singapore, March 2006 *BG Englert*

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