**Problem 1** (25=7+18 marks)
A point mass $m$ is thrown upwards from $r(t = 0) = 0$ with initial velocity $v(t = 0) = v_0 e_z$, $v_0 > 0$, and then moves under the influence of its weight $mg = -mg e_z$ and the Newtonian frictional force $-m \gamma v$. The point mass is highest above ground at time $t_1$, and it is back at the initial height $z = 0$ at time $t_2$.

(a) Explain by qualitative arguments, why $t_2 > 2t_1$, that is: it takes longer to fall down than to fly up.

(b) Find expressions that relate $t_1$ and $t_2$ to $\gamma v_0 / g$ and then use them to demonstrate that $t_2 > 2t_1$ is indeed the case. — Hint: $\sinh(\vartheta) > \vartheta$ for $\vartheta > 0$.

**Problem 2** (45=5+8+12+4+16 marks)
A point mass $m$ is moving along the $x$ axis under the influence of the force associated with the potential energy

$$V(x) = F(\sqrt{|x| + a} - \sqrt{a})^2$$

with constants $F > 0$ and $a > 0$.

(a) What are the metrical dimensions of $F$ and $a$?

(b) Which combination of $m$, $F$, and $a$ has the metrical dimension of energy? Which combination has the metrical dimension of time?

(c) Which simple expressions approximate $V(x)$ for $|x| \ll a$ and $|x| \gg a$?

(d) What is the period of small-amplitude oscillations?

(e) What is the energy-dependent period $T(E)$ for motion between two turning points? — Hint: The substitution $x = (y^2 + 2y)a$ could be helpful.

**Problem 3** (30=15+15 marks)
Consider the following two force fields:

(i) \[ F \equiv \lambda \begin{pmatrix} y^2 + yz \\ 2xy - z^2 + xz \\ 2yz + xy \end{pmatrix} \quad \text{with} \quad \lambda = \text{constant}; \]

(ii) \[ F = a \times (r \times a) \quad \text{with} \quad a = \text{constant}. \]

(a) Is force (i) conservative? If yes, find a potential energy for it.

(b) Is force (ii) conservative? If yes, find a potential energy for it.