**Problem 1** (15 marks)
Point masses $m_1, m_2, \ldots, m_J$ have conservative line-of-sight pair forces among them and are exposed to the external forces $F_j^{(\text{ext})} = m_j g$ where $g$ is the same gravitational acceleration for all masses. At time $t_0$, the initial conditions give values $R_0$, $P_0$, $E_0$, and $L_0$ to the center-of-mass position $R$, the total momentum $P_{\text{tot}}$, the total energy $E_{\text{tot}}$, and the total angular momentum $L_{\text{tot}}$. Find $R(t)$, $P_{\text{tot}}(t)$, $E_{\text{tot}}(t)$, and $L_{\text{tot}}(t)$.

**Problem 2** (20 marks)
A point mass is scattered elastically by an impenetrable sphere with radius $R$. Invoke “angle of reflection = angle of incidence” and so determine the relation between the scattering angle $\theta$ and the impact parameter $b$. Then find the differential scattering cross section $d\sigma$ and the total cross section $\sigma = \int d\Omega \frac{d\sigma}{d\Omega}$.

**Problem 3** (20 marks)
Under the gravitational pull $g = ge_y$, a point mass is moving along the brachistochrone to get from $(x_1, y_1) = (0, 0)$ to $(x_2, y_2) = (a, 0)$. What is the average speed?

**Problem 4** (25=10+8+7 marks)
Point mass $m$ is moving along the horizontal $x$ axis. A spring of natural length $a$ and spring constant $k$ connects the mass to point $(0, a)$ on the $y$ axis.

(a) State the Lagrange function $L(t, x, \dot{x})$ and derive the equation of motion for $x(t)$. Is there a conserved quantity?

(b) For the parameterization $x = a \sinh \vartheta$, state the Lagrange function $L(t, \vartheta, \dot{\vartheta})$, and derive the equation of motion for $\vartheta(t)$.

(c) Which approximate equations of motion apply for $|x| \ll a$ and $|\vartheta| \ll 1$?