Problem 1 (10 marks)
Consider a collection of charges in nonrelativistic motion, so that
\[
\vec{G}_{\text{ch}}(\vec{r}, t) = \sum_j \delta(\vec{r} - \vec{r}_j(t)) m_j \vec{v}_j(t),
\]
\[
\vec{T}_{\text{ch}}(\vec{r}, t) = \sum_j \delta(\vec{r} - \vec{r}_j(t)) m_j \vec{v}_j(t) \vec{v}_j(t)
\]
are their momentum density and momentum current density, respectively. Show that
\[
\frac{\partial}{\partial t} \vec{G}_{\text{ch}} + \nabla \cdot \vec{T}_{\text{ch}} = \vec{f},
\]
where \(f(\vec{r}, t)\) is the familiar Lorentz force density. How does the local momentum conservation follow from this?

Problem 2 (30 marks)
Observer A uses unprimed coordinates, observer B uses primed coordinates. They move relative to each other with velocity \(\vec{v}\). By considering both (i) infinitesimal Lorentz transformations and (ii) a finite Lorentz transformation, show that both observers see the same total charge, that is
\[
\int (d\vec{r}) \rho(\vec{r}, t) = \int (d\vec{r}') \rho'(\vec{r}', t'),
\]
where the integrations cover all of space.
Hints: You can choose \(\vec{v}\) along the \(z\) axis if you like; the identity
\[
f(x - a) = f(x) - \int dx' \left[ \eta(x - x') - \eta(x - a - x') \right] \frac{df(x')}{dx'}
\]
could be useful.

Problem 3 (20 marks)
A point dipole \(\vec{d}(t)\) moves along the trajectory \(\vec{R}(t)\), so that the charge density is given by
\[
\rho(\vec{r}, t) = -\vec{d}(t) \cdot \nabla \delta(\vec{r} - \vec{R}(t)).
\]
Verify that \(\int (d\vec{r}) \rho(\vec{r}, t) = 0\) and \(\int (d\vec{r}) \vec{r} \rho(\vec{r}, t) = \vec{d}(t)\). Then find the corresponding current density \(\vec{j}(\vec{r}, t)\) and express the magnetic dipole moment \(\vec{\mu}(t)\) in terms of \(\vec{d}(t)\) and \(\vec{R}(t)\).