Problem 1 (15 marks)
Your friend told you that \( u(x, y) = \cos x \cosh y \) is the real part of an entire function. Determine the imaginary part \( v(x, y) \). Then write the function compactly.

Problem 2 (15 marks)
The values \( f(z) \) of a certain entire function are real for all real values of \( z \). Show that \( f(z^*) = f(z)^* \). [Hint: Consider the Taylor expansion at \( z = 0 \).]

Problem 3 (20 marks)
Evaluate
\[
\int_{z_1}^{z_4} dz \ |z|^2
\]
along this path:
(1) first follow the straight line from \( z_1 = -1 \) to \( z_2 = -r \) with \( r > 0 \);
(2) then follow either one of the two half-circles connecting \( z_2 = -r \) with \( z_4 = r \);
(3) finally follow the straight line from \( z_3 = r \) to \( z_4 = 1 \).

Problem 4 (20 marks)
For which values of \( z \) has
\[
f(z) = \frac{e^{-iz}}{(z - 2)(z^2 - 4)}
\]
singularities? What are the residues of \( f(z) \) at those singularities?

Problem 5 (30 marks)
For real \( t \neq 0 \), evaluate
\[
\int_0^{2\pi} d\varphi \ \frac{\cosh t}{2\pi \sinh t + i \sin \varphi}
\]
with the aid of the residue method.