
PC2232 Physics for Electrical Engineers

Single Slit Diffraction

Objectives

- To investigate the diffraction patterns produced by monochromatic laser light for various slit configurations..
- To determine the wavelength of the laser light from measurements of the intensity distribution of light for the single slit.
- To determine the thickness of a human hair strand from the diffraction pattern.

Equipment

- Single slit wheel with holder, aperture disk, diode laser, multiple slit wheel with holder
- Computer, interface device, Data Studio software
- Light sensor, motion sensor, precision linear translator, optical bench
- Micrometer screw gauge

Theory

When coherent light of monochromatic wavelength is incident upon a slit, the light diverges as it passes through the slit in a process known as *diffraction*. A laser produces coherent light, which means all the light striking the slit is in phase. If the light then falls on a screen placed at a large distance from the slit, it produces a pattern of alternating bright and dark images of the slit. This pattern is referred to as a Fraunhofer diffraction pattern, which is the simplest case of diffraction. It occurs when rays emerging from the slit can be considered to be parallel.

The diffraction process is explained by the fact that light is a form of electromagnetic wave and the different portions of the slit behave as if they were separate sources of light waves – Huygens’ principle. At each point on the screen, the light from the different portions of the slit will have a different phase due to the different path length of light from each portion of the slit to the point on the screen. Light from the different portions of the slit will interfere with each other and the resultant intensity will vary at different places on the screen. This is illustrated in Figure 1 where five rays are shown from different portions of the slit. The choice of five is arbitrary and the slit could be divided into any number of portions.

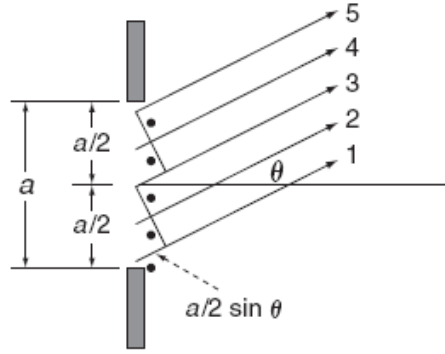


Figure 1: Path of five rays from different portions of the slit.

Consider wave 1 from near the bottom of the slit and wave 3 from the center of the slit. They differ in path length by the amount $(a \sin \theta)/2$ as shown, where a is the width of the slit and θ is the angle each of the rays makes with the horizontal. It is also true that the path difference between rays 2 and 4 is $(a \sin \theta)/2$. If this path difference is exactly equal to half a wavelength, i.e., $\lambda/2$, the two waves will be 180° out of phase and will interfere destructively and cancel each other. Therefore, waves from the upper half of the slit will be 180° out of phase with waves from the lower half of the slit when

$$\frac{a}{2} \sin \theta = \frac{\lambda}{2} \quad (1)$$

By dividing the slits into $2m$ portions, it can be shown that the condition for destructive interference will be satisfied at angles θ on the screen above and below the center of the pattern given by

$$\sin \theta = m \frac{\lambda}{a} \quad (m = \pm 1, \pm 2, \pm 3, \dots) \quad (2)$$

There is no simple expression for the location of the maxima on the screen other than that for the principal maximum at the center of the pattern. The other maxima are much less intense than the principal maximum and are located approximately halfway between the minima. The diffraction pattern that appears on a screen will have an intensity variation above and below the slit as shown in Figure 2.

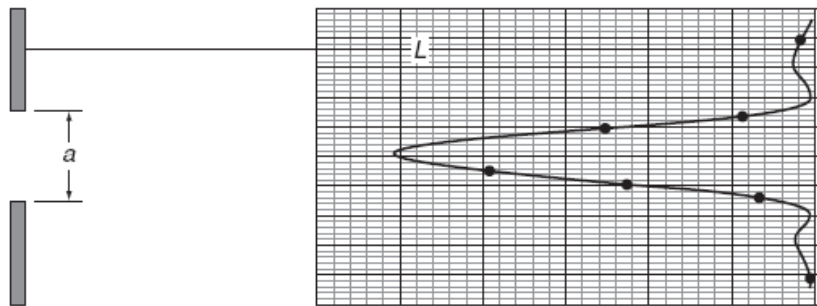


Figure 2: Diffraction pattern on a screen distance L from a single slit with width a .

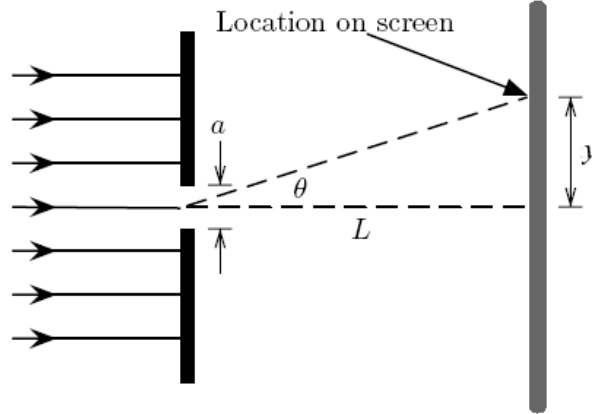


Figure 3: A schematic diagram for the light diffraction setup.

To study diffraction of light, laser light is passed through a narrow single slit and the diffraction pattern is formed on a distant screen. An imaginary reference line is drawn perpendicularly from the center of the slit out to the screen (see Figure 3), which is a distance L away. The intensity variation of the diffraction pattern can then be measured accurately as a function of the distance y from the reference line. In the theoretical description of the diffraction pattern, however, it is more convenient to quantify the light intensity as a function of the sine of the angle θ defined accordingly by

$$\sin \theta = \frac{y}{\sqrt{y^2 + L^2}} \quad (3)$$

The theory of diffraction predicts that the spatial pattern of light intensity on the viewing screen by a light wave passing through a single rectangular-shaped slit is given by

$$I(\theta) = I_0 \left[\frac{\sin \left(\frac{\pi a \sin \theta}{\lambda} \right)}{\frac{\pi a \sin \theta}{\lambda}} \right]^2 \quad (4)$$

where I_0 is the light intensity at $\theta = 0^\circ$ and the quantities in parentheses are in radians. As a first step in understanding the meaning of Equation (4), its prediction for the diffracted wave's intensity I versus angle θ is plotted in Figure 4. In these sequence of plots, the choice of the slit width a is progressively narrowed, starting first with a width equal to 100 times the size of the incident wave's wavelength, then decreased to 10, 1, and, finally, 0.1 times the wavelength. From these plots, it is apparent that, when the wavelength is small compared to the slit width (e.g. $a = 100\lambda$), the diffracted wave's energy is concentrated in a very small angular spread. In this case, diffraction occurs negligibly. However, as the wave's wavelength becomes comparable in size to the width of the slit (e.g. $a = 10\lambda$ and $a = \lambda$), the angular spread of the diffracted wave's energy becomes significant, and when $a = 0.1\lambda$, the diffracted wave's energy is almost uniformly spread over all angles from $\theta = 0^\circ$ to $\theta = 90^\circ$.

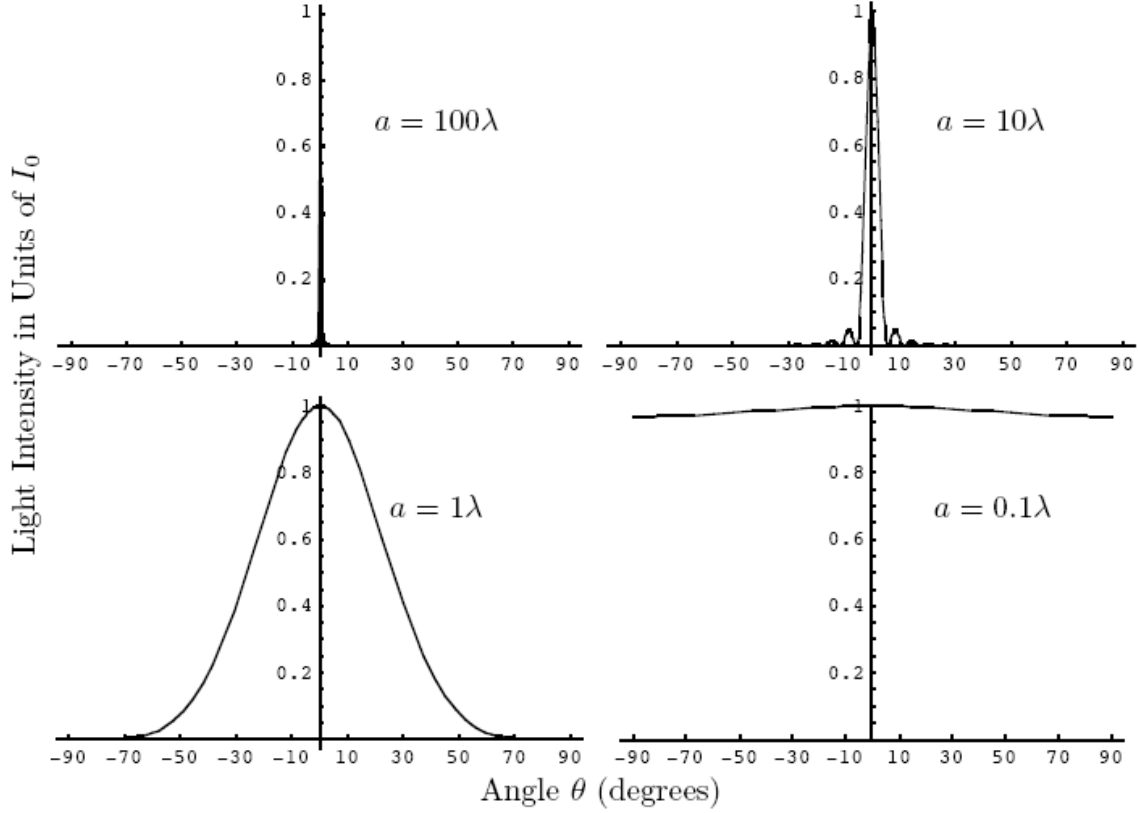


Figure 4: Diffraction of light as a function of the ratio λ/a .

To understand the generic features on the intensity variation (4) without having to make a particular choice for the slit width a , it is convenient to rewrite this expression in terms of the quantity α :

$$\alpha \equiv \frac{a \sin \theta}{\lambda} \quad (5)$$

Then, equation (4) becomes

$$I(\theta) = I_0 \left[\frac{\sin(\pi\alpha)}{\pi\alpha} \right]^2 \quad (6)$$

In the above expression, the argument of the sine function is in radians.

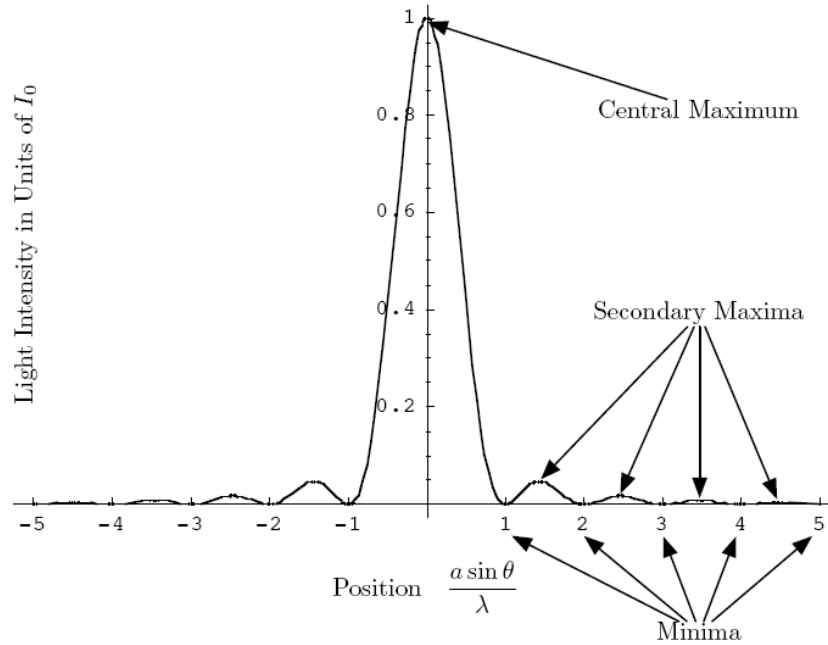


Figure 5: Diffraction of light as a function of the ratio λ/a .

The plot of equation (6) is shown in Figure 5. It can be seen that the diffraction pattern formed when light wave passes through a rectangular-shaped slit consists of a set of bright spots (the principal maximum and many secondary maxima) interspersed with regions of darkness (the minima). A summary of the key features of this rectangular-slit diffraction pattern is as follows:

- **Minima.** The minima (locations of zero light intensity) occur at the angle θ given by $\alpha = a \sin \theta / \lambda = \pm 1, \pm 2, \pm 3, \pm 4 \dots$ and are called first, second, third, fourth, \dots minima, respectively. It is to be noted that the condition for minima is the same as obtained in (2) from a simpler geometrical argument.
- **Principal Maxima.** The central peak, bracketed by the two “first minima” (which are located at $\alpha = a \sin \theta / \lambda = \pm 1$), is the region of highest light intensity and most of the diffracted wave’s energy is concentrated in this region.
- **Secondary Maxima.** A detailed analysis of equation (6) (which involves taking derivatives to find the maxima of this expression) reveals that the secondary maxima occur at angles θ given by $\alpha = a \sin \theta / \lambda = \pm 1.43030, \pm 2.45902, \pm 3.47089, \pm 4.47741$ and are called the first, second, third and fourth secondary maxima are only 4.7%, 1.6%, 0.8%, 0.5% respectively of I_0 (the intensity of the principal maximum).

Laboratory Work

Apparatus and Computer Setup

1. Mount the single-slit wheel with holder to the optics bench. To select the desired slits, rotate the disk until it clicks into place with the desired slits at the center of the holder.

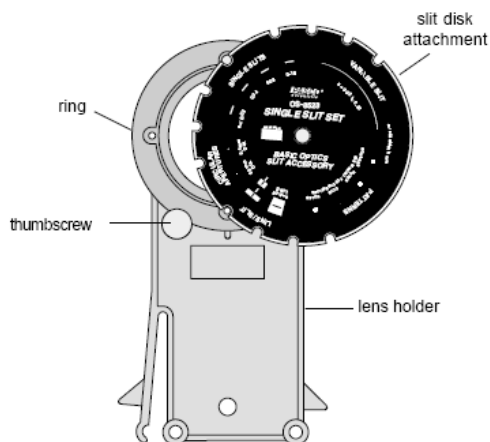


Figure 6: Slit accessory and lens holder.

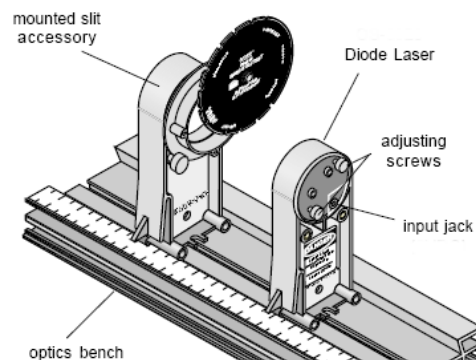


Figure 7: Using the slit accessory with the diode laser.

2. Align the laser beam with the slit.
 - Mount the diode laser at one end of the bench. Put the single-slit wheel with holder on the optic bench a few centimeters away from the laser with the disk side of the holder closer to the laser. Plug in the laser and turn it on.
 - Adjust the position of the laser beam from left-to-right and up-and-down until the beam is more or less centered on the slit. The screws to do this are on the back of the diode laser (see Figure 7). Once this position is set, it is not necessary to make any further adjustments of the laser beam when viewing any slits on the disk. When you rotate the disk to a new slit, the laser beam will already be aligned. The slits click into place so you can easily change from one slit to the next even in the dark.
3. Prepare the rotary motion sensor and the light sensor.
 - Mount the rotary motion sensor in the rack of the linear translator. Then mount the linear translator to the end of the optics bench so the rack is perpendicular to the optics bench (see Figure 8). The teeth on the rack engage a gear inside the rotary motion sensor, causing the gear to rotate when the rotary motion sensor moves along the rack. The rotary motion sensor measures its linear position along the rack. (This part should already be set up by previous students or the lab officers and you may proceed to the next step.)
 - The light sensor with the aperture bracket is mounted on the rotary motion sensor rod clamp. The light sensor contains a silicon photodiode. When illuminated by light of a given intensity, it will output a voltage that is proportional

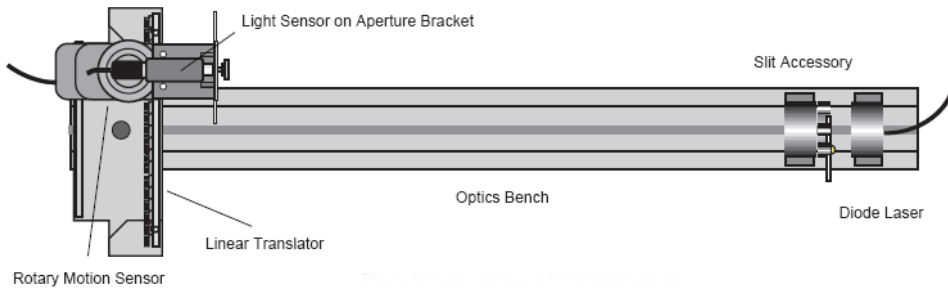


Figure 8: Setting up the equipment.

to the light intensity. At the side of the light sensor, there is three buttons which select the electronic amplification factor. Set the gain to appropriate levels for different parts your experiment (usually 0 – 100 for obtaining the full pattern without “clipping”, and 0 – 1 for measuring the intensity of secondary maxima).

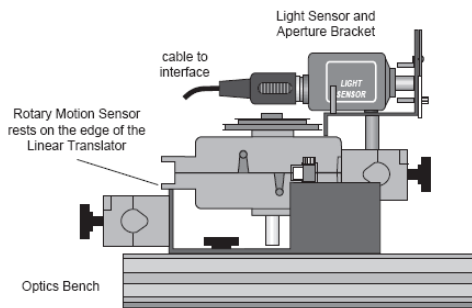


Figure 9: Light sensor and aperture bracket.

Note: Since the detector area of silicon photodiode is fairly large (about 8 mm in diameter), an aperture disk is placed in front of the sensor to restrict the part of the light pattern that would hit the photodiode. The aperture disk has several slit apertures labeled 1 through 6 with the following widths: 1 = 0.1 mm, 2 = 0.2 mm, 3 = 0.3 mm, 4 = 0.5 mm, 5 = 1.0 mm and 6 = 1.5 mm.

4. Prepare the computer interface and software settings

- Connect the *Powerlink* interface to the computer, turn on the interface and the computer.
- Connect the light sensor and rotatory motion sensor to the interface if necessary.
- Launch Data Studio.
- Select the **Rack and Pinion** option in the **Setup** section for the rotatory motion sensor to measure the **Linear Position** of the sensor directly.
- The default rate of measurement (data acquisition) is 10 Hz. Set this to a higher rate if necessary.
- Reset the graph to choose Intensity for the y -axis, and linear position for the x -axis.

Part A: Single Slit Diffraction Pattern (Qualitative)

In this part of the experiment, you will observe qualitatively the single slit diffraction pattern due to a wide and narrow slits respectively.

- A-1. Rotate the single slit wheel until the laser light beam is incident on the variable slit. By rotating the disk, the laser light passes through a slit whose width can be continuously varied from $a = 0.20$ mm to $a = 0.02$ mm. Position the disk so that the light is passing through the widest portion of the variable slit where $a = 0.20$ mm. Use the adjustment screws on the back of the diode laser to adjust the vertical position of the beam if necessary. Rotate the single slit disk so that the laser passes through a narrower and narrower portion of the variable slit. Observe the diffraction pattern on the white screen of the aperture bracket (see Figure 10). Note the spatial width and overall brightness of the central peak as the slit becomes narrower.

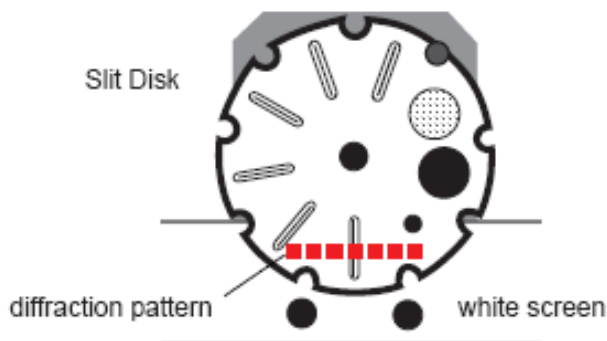


Figure 10: Diffraction pattern on the white screen.

- A-2. Identify the central peak, which is the brightest region of light at the center of the pattern and is bracketed by two dark spots (the first minima). Also note the much dimmer secondary maxima symmetrically located on both sides of the central maxima and separated from each other by the second, third, fourth, . . . minima.

Part B: Single Slit Diffraction Pattern (Quantitative)

In this part of the experiment, you will quantitatively examine the diffraction pattern produced when laser light beam ($\lambda = 650 \text{ nm}$) passes through a rectangular shaped single slit with slit width $a = 0.16 \text{ mm}$. In particular, you will examine in details the angular positions and intensities for the central peak and secondary maxima. The angular positions for the minima will also be examined.

- B-1. Rotate the single slit wheel until the laser light beam is incident on the rectangular shaped single slit with slit width $a = 0.16 \text{ mm}$.
- B-2. Rotate the aperture disk on the front of the aperture bracket until the narrowest slit opening (slit labeled 1) is in front of the light sensor opening. You may choose other settings if that improves your results.
- B-3. Move the light sensor to one side of the diffraction pattern so that very little light is entering the sensor.
- B-4. Press the “Start” button (in the software) to begin taking data. Then, slowly and smoothly move the sensor across the pattern by rotating the large pulley of the rotary motion sensor so that the light sensor can record the diffraction pattern. Click the “Stop” button when the sensor is moved sufficiently far to the other side of the diffraction pattern.
- B-5. Press the “Scale to fit” button (leftmost button in graph toolbar) to rescale the plot and have a better view of your data. Ensure that the central fringe is not “clipped”. Otherwise, choose a lower sensitivity for the sensor and repeat the scan.
- B-6. On the graph, use the “Smart Tool” in graph toolbar to measure the intensity of the central peak. Record it as I_0 in Data Table 2.
- B-7. Locate first secondary maxima (two of them appear symmetrically to the right and left of the central peak) on the graph. Use “Smart Tool” to measure the distance for each first secondary maxima from the central peak respectively and record them in Data Table 2. Also, measure and record their intensities in Data Table 2.
- B-8. Repeat step B-7 for the second, third and fourth secondary maxima.
- B-9. Locate first minima (two of them appear symmetrically to the right and left of the central peak) on the graph. Use “Smart Tool” to measure the distance for each first minima from the central peak respectively and record them in Data Table 3.
- B-10. Repeat step B-9 for the second, third, fourth and fifth minima.
- B-11. Cancel all zooms and fix up the graph window so that all data collected can be seen. Print the graph and label each secondary maxima and minima on both sides of the central peak. Title this graph “Single Slit Diffraction Pattern ($a = 0.16 \text{ mm}$)”.

Part C: Diffraction Pattern of A Human Hair

Diffraction of light may be used to determine the thickness of a human hair. When a coherent beam of light is shone on a single strand of hair, the beam diffracts around the edges of the hair and casts a diffraction pattern. The interference pattern cast by hair shares some similar features with that of a narrow single slit. The same equation describing diffraction minima for the single slit can be used to describe the diffraction minima for a strand of human hair. In this part of the experiment, the thickness of your hair strand will be estimated by examining the positions of the minima of its diffraction pattern.

- C-1. Pull or cut a strand of hair from your head and mount it vertically and directly in front of the laser beam.
- C-2. Adjust the position of the hair or the laser beam if necessary so that the beam is centered on the strand of hair.
- C-3. Determine the distance from the strand of hair to the screen and record the hair-to-screen distance as L in Data Table 4.
- C-4. Obtain a diffraction pattern using similar procedure as in Part B. Concentrate on the minima of the pattern.
- C-5. Locate first minima (two of them appear symmetrically to the right and left of the central peak) on the graph. Use “Smart Tool” to measure the distance for each first minima from the central peak respectively and record them in Data Table 4.
- C-6. Repeat the previous step for the second, third, fourth and fifth minima (if still visible).
- C-7. Print the graph and label each secondary maxima and minima on both sides of the central peak. Title this graph “Single Slit Diffraction Pattern (Human Hair Strand)”.
- C-8. Tape about 2 cm of the hair used for the experiment to the graph..
- C-9. Measure and record the thickness of the hair strand as d in Data Table 4. You may use the micrometer screw gauge provided or any other method that you think can give better results. Describe briefly your method if you do not use the micrometer screw gauge.

After you have finished taking all the necessary measurements, you may want to spend some time examining the diffraction/interference patterns of the other configurations, including those on the multiple slit wheel. Your report will not include this – it is meant to complement what you have learnt and seen in the lectures.