INTRODUCTION

A beam of potassium atoms generated in a hot furnace travels along a specific path in a magnetic two-wire field. Because of the magnetic moment of the potassium atoms, the nonhomogeneity of the field applies a force at right angles to the direction of their motion. The potassium atoms are thereby deflected from their path.

By measuring the density of the beam of particles in a plane of detection lying behind the magnetic field, it is possible to draw conclusions as to the magnitude and direction of the magnetic moment of the potassium atoms.

THEORY

The potassium atoms forming the atomic beam have one outer electron in the ground state 4s. The orbital angular momentum is equal to zero. The magnetic moment $\vec{\mu}$ of the potassium atom due to this electron is therefore attributable only to its spin $S$.

$$\vec{\mu} = -\frac{e}{2m_0} g_S \vec{S}.$$  

If one considers the component $S_z$ of the spin in a given z-direction, the system has two different possible orientations, characterized by the quantum numbers

$$m_s = \pm \frac{1}{2}.$$  

The z-component of spin takes the eigenvalue

$$S_z = m_s \hbar.$$  

The associated magnetic moment in the z-direction take the value

$$\mu_z = -\frac{e\hbar}{2m_0} m = -\mu_B m.$$
with the Bohr magneton

$$\mu_B = -\frac{e\hbar}{2m_e} = -9.248 \cdot 10^{-24} \text{ A m}^2$$

and

$$m = m_s \cdot g_s .$$

The literature value of the g-factor is

$$g_s = 2.0024 . \quad (2)$$

Hence,

$$m = \pm 1.0012 \approx \pm 1 . \quad (3)$$

The object of the Stern-Gerlach experiment is to establish the directional quantization of the electron spin. Furthermore, according to which quantity is taken as known, the value of $\mu_z$; $\mu_B$ m or $g_s$ can be determined.

The forces acting on the potassium atom are attributable to their magnetic moment and arise when the field is inhomogeneous:

$$F_z = \mu_z \frac{\partial B}{\partial z} = -m\mu_B \frac{\partial B}{\partial z} . \quad (4)$$

To produce an inhomogeneous magnetic field which simulates the field produced by two parallel wires separated by $2a$, the specially shaped pole-pieces (made of magnetic soft iron) shown in Fig. 1 is used.

![Fig. 1 Two wire field.](image-url)
The following expression applies to the two-wire field

\[
\frac{\partial B}{\partial z} = \frac{1}{0.968} \frac{B}{a}
\]  

(5)

where the magnetic introduction $B$ against the excitation current $i$ can be calibrated. (See Appendix.)

The velocity $v$ of the potassium atoms entering the magnetic field can be considered with sufficient accuracy as being along the same direction (x-direction) before entry into the field. The following transit in the x-direction should be borne in mind (Fig. 2): a time

$$\Delta t = \frac{L}{v}$$

for passing through the magnetic field of length $L$ and a time

$$t = \frac{\ell}{v}$$

for the distance $\ell$ from the point of entry into the magnetic field to the point of entry into the plane of the detector.

Because of the effectively constant force in the z-direction, the potassium atoms of mass $M$ acquire, by virtue of the inhomogeneity of the field, a momentum

$$\Delta z = F_z \Delta t = F_z \cdot \frac{L}{v} = - \frac{m_B L}{v} \frac{\partial B}{\partial z} .$$

![Fig. 2](image)

**Fig. 2** Particle track between magnetic analyzer and detection plane.
It follows that the point of impact \( u \) of a potassium atom of velocity \( v \) in the \( x \)-direction, at a given field inhomogeneity, is

\[
u = z + \frac{1}{2} z \Delta t + z(t - \Delta t) = z + \left(1 - \frac{1}{2} \frac{L}{t} \right) \frac{\Delta}{v} z
\]

where

\[
\frac{1}{2} z \Delta t
\]

is the path element covered by a potassium atom immediately after passing through the magnetic field in the \( z \)-direction. Hence, there is the following fundamental relationship between the deflection \( u \), the particle velocity \( v \) and the field inhomogeneity \( \partial B/\partial z \).

\[
u = z - \frac{\mu B L}{Mv^2} (1 - \frac{1}{2} \frac{L}{t}) m\mu_B \frac{\partial B}{\partial z}
\]

(6)

where

\[
u - z > 0 \text{ for } m = -1
\]

and

\[
u - z < 0 \text{ for } m = +1
\]

It will be noticed that the faster particles are deflected less from their path than the slower ones.

In order to produce a beam of potassium atoms of higher average particle velocity, a furnace heated to a defined temperature \( T \) is used. In the furnace, the evaporated potassium atoms are sufficiently numerous to acquire a maxwellian velocity distribution, i.e., the number of atoms with a velocity between \( v \) and \( v + dv \) in each elementary volume \( dV \) of the furnace is proportional to

\[
e^{-\frac{mv^2}{2kT}} \cdot v^2 dv
\]

This proportionality applies also when one considers only the velocity directions lying within a solid angle \( d\Omega \) which is determined at \( x = 0 \) by tracks of width \( dz \). The atoms which emerge from the opening in the furnace, and which have entered the magnetic field between \( z \) and \( z + dz \), with a
velocity between $v$ and $v + dv$, obviously satisfy a velocity function involving the third power of $v$ (Fig. 3)

$$d^2n = \frac{\phi_m(z) e^{-\frac{Mv^2}{2kT}} \cdot v^3 dv \cdot dz}{\int_0^\infty e^{-\frac{Mv^2}{2kT}} \cdot v^3 dv}$$ (7)

Thus, only those potassium atoms traverse the strip $dz$ in time $dt$ with velocity $v$ at a later point in time corresponding to the transit time, which come from volume element $dV$ existing in a region at depth $vdt$ behind the opening in the furnace. The volume of this region is proportional to $v$ and contributes likewise to the distribution function. Indexing with $m$ takes account of the two possible magnetic moments of the potassium atoms. On symmetry grounds it can be assumed that both directional orientations are equally probable.

The function $\phi_m(z)$ represents the spatial profile of the numbers of particles, for particles of orientation $m$ at the position $x = 0$. It arises through limitation of the atomic beam by appropriate systems of diaphragms. The function $\phi_m(z)$ differs from zero within a rectangular area of width $D$ (beam enclosure).

The object of the following calculation is to calculate the particle current density $I$ in the measuring plane $x = 0$, as a function of the
position $u$, from the distribution which depends on $v$ and $z$, since this
density is proportional to the signal at the detector. All potassium atoms
entering the magnetic field at a height of $z$ are spread by an amount $du$ at
position $u$ on account of their differences in velocity $dv$. For equal values
of $z$, therefore, the following conversion from $v$ to $u$ applies:

$$ v^3 dv = \frac{1}{4} \left| \frac{\partial v^4}{\partial u} \right| du . $$

From equation (6) for the deflection of the path as a function of $v$, one
obtains by substitution and differentiation:

$$ v^3 dv = \frac{1}{2} \left( \frac{I.L \left(1 - \frac{1}{2} I\right) \mu_B}{2M} \frac{\partial B}{\partial z} \right)^{2} \frac{du}{|u - z|^3} $$

In addition,

$$ \frac{Mv^2}{2kT} = \frac{I.L \left(1 - \frac{1}{2} I\right) \mu_B}{2kT \left|u - z\right|} \frac{\partial B}{\partial z} $$

If, to abbreviate, we now let

$$ q = - \frac{I.L \left(1 - \frac{1}{2} I\right) \mu_B}{2kT} \frac{\partial B}{\partial z} $$

and

$$ n_0 = - \left[ I.L \left(1 - \frac{1}{2} I\right) \mu_B \frac{\partial B}{\partial z} \right]^{2} \frac{Mv^2}{16M^2} \int_{0}^{\infty} e^{-\frac{Mv^2}{2kT}} v^3 dv $$

we obtain from (7) the distribution

$$ d^2n = n_0 \phi_m(z) \cdot e^{-\frac{q}{\left|u - z\right|}} \cdot \frac{du}{\left|u - z\right|^3} dz . $$
We now integrate with respect to $z$ and sum over the possible orientations $m$, and so derive the desired particle current density at position $u$:

$$I = \sum_m^{+D}_{-D} \int d^2n \frac{+D}{du} = n_0 \int_{u-z>0}^{-D} -D \phi_{-1/2}(z) e^{-q |u-z|} \frac{dz}{|u-z|^3}$$

$$+ n_0 \int_{u-z<0}^{-D} -D \phi_{+1/2}(z) e^{-q |u-z|} \frac{dz}{|u-z|^3}$$

By reason of the equivalence to the particle profile of the orientations $m = -1/2$ and $m = +1/2$,

$$\phi_{+1/2}(z) = \phi_{-1/2}(z) = I_0(z)$$

and hence

$$I = n_0 \int_{-D}^{+D} I_0(z) e^{-q |u-z|} \frac{dz}{|u-z|^3} (10)$$

For a vanishing magnetic field or a vanishing inhomogeneity $u = z$ and is independent of $v$. In this case, the particle current density in the measuring plane is defined as $I_0(u)$.

The course of the particle current density $I(u)$ depends, among other factor, on how $I_0(u)$ is formed. As the simplest approximation, one can start from a beam enclosure of any desired narrowness:

$$I^{(0)}_0(z) = 2D I_0 \delta(z)$$

(11)

We then get

$$I^{(0)}_0(u) = 2Dn_0 I_0 \int_{-D}^{+D} \delta(z)e^{-q \frac{u-z}{|u-z|^3}} \frac{dz}{|u-z|^3}$$

and therefore

$$I^{(0)}_0(u) = 2Dn_0 I_0 \frac{e^{-q \frac{u}{|u|^3}}}{|u|^3}. (12)$$
The particle current density $I^{(0)}(u)$ for narrow beam profiles is therefore proportional to the width $2D$ determined by the diaphragm system. The position $u_e$ of the intensity maximum is found by differentiating with regard to $u$:

$$\frac{dI^{(0)}(u)}{du} = 2Dn_0I_0 \frac{q - 3|u|}{u^5} e^{-\frac{q}{|u|}}.$$  

From

$$\frac{dI^{(0)}(u)}{du} (u_e^{(0)}) = 0$$  

we obtain the determining equation for $u_e^{(0)}$:

$$u_e^{(0)} = \pm \frac{1}{3} q = \pm \frac{I.L \left(1 - \frac{1}{2} \frac{L}{I}\right) \mu_B \frac{\partial B}{\partial z}}{6kT}.$$  

The distance of the maxima from the x-axis (beam deflection) therefore increase in proportion to the field inhomogeneity.

A better compatibility of the calculation with the experiment is achieved by regarding the beam enclosure with width $2D$ as being infinitely long, and by describing the beam profile as two steep straight lines with a parabolic apex (Fig. 4).

$$I_0(z) = i_0 \begin{cases} 
D + z & -D \leq z \leq -p \\
D - \frac{1}{2} p - \frac{1}{2} \frac{z^2}{p} & -p \leq z \leq +p \\
D - z & +p \leq z \leq +D 
\end{cases}$$

$$\frac{dI_0}{dz} = i_0 \begin{cases} 
1 & -D \leq z \leq -p \\
-\frac{z}{p} & -p \leq z \leq +p \\
-1 & +p \leq z \leq +D 
\end{cases}$$

$$\frac{d^2I_0}{dz^2} = i_0 \begin{cases} 
0 & -D \leq z \leq -p \\
\frac{1}{p} & -p \leq z \leq +p \\
0 & +p \leq z \leq +D 
\end{cases}$$
In this model, $I_0(z)$ is regarded as being twice differentiable. The particle current density $I(u)$ based on this supposition has values, dependent on the inhomogeneity of the magnetic field and hence on $q$, which have maxima at positions $u_e(q)$, which differ to a greater or lesser extent from the positions $u_e^{(0)} = \pm \frac{q}{3}$ resulting from the approximation assuming an infinitesimally narrow beam enclosure.

It can be shown that

$$q = 3u_e - \frac{p^4 - \frac{1}{5} p^4}{p^2 - \frac{1}{3} p^2} \cdot \frac{1}{u_e}$$

$$= 3u_e - \frac{c}{u_e}$$

(16)

where $$c = \frac{D^4 - \frac{1}{5} p^4}{D^2 - \frac{1}{3} p^2}$$

For sufficiently large inhomogeneous fields

$$u_e = \frac{q}{3} + \frac{c}{q} .$$

Using equations (8) and (16), the Bohr magneton $\mu_B$ is given by:

$$\mu_B = \frac{2kT}{I. L \left(1 - \frac{1}{2} \frac{L}{I}\right)} \cdot \frac{3u_e - \frac{c}{u_e}}{\frac{\partial B}{\partial z}}$$

(17)
EXPERIMENTAL SET-UP

Fig. 5 shows the experimental set-up for the Stern-Gerlach apparatus on pumping unit with connected power supply and measuring instruments.

Fig. 5  Experimental set-up; Stern-Gerlach apparatus with high-vacuum pump stand.
The schematic arrangement of the set-up without adjusting device is shown in Fig. 6.

Fig. 6  Schematic arrangement of the experimental set-up 
(without adjusting devices).

A beam of potassium is produced by heating the metal in a small electric furnace and then defined as a thin beam by means of slits. The magnetic deflecting field produced between the specially shaped pole-pieces of an electromagnet simulates a two-wire field.

The detector is a surface ionization detector; it can be swivelled by means of a level system and enables the intensity distribution in the beam to be measured electrically with good sensitivity. The vacuum container is provided with bellows.
The principle and arrangement of the adjusting device are shown in Fig. 7.

View from the detector the beam always emanates from the center of magnet. In order to scan the intensity of beam cross-section, the detector has to be guided on a circular curve, the center of which is located at the center of the magnet. This is achieved by means of a level guide. Since there is no reference point available near the detector for measuring the angle of adjustment, the level is extended beyond its pivot in the direction of the beam source where it is adjusted with the aid of a spindle; the latter is provided with a scale having 100 graduations. One revolution \( u \) of the spindle corresponds to an adjusting path \( S_D \) of 1 mm (10 revolutions possible).

The leverage is 1:1.8, i.e. one revolution of the spindle scale corresponding to a displacement \( S_D = 1.8 \) mm of the detector.
The electrical circuit in the Stern-Gerlach experiment is shown in Fig. 8.

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**Fig. 8  Electrical circuit.**

**Procedure**

1. Evacuating the Stern-Gerlach apparatus and bringing the furnace to the desired temperature (~180 °C) take several hours. Ask the technologist in charge of the experiment to prepare the apparatus in the morning so that you can perform the experiment in the afternoon, on the same day.

2. Before carrying out the measurement, demagnetize the electromagnet. This is done by reducing the excitation current in steps by small amounts, reversing the polarity at each step using the commuting switch in the circuit shown in Fig. 8.
3. Using the following settings,
   Furnace temperature: ~180 °C
   Detector voltage: 11 V
   run the Step Motor Software to obtain the graph of the detector ionization current \(i_I\) as a function of detector displacement \(U_e\).

4. Software procedures
   I. Measurement

   i. Open the “Measure” program

   ii. Click the “Gauge” menu. Confirm that “Step motor” has been selected

   iii. Go to “file” menu and click “Open measurement” to display the following window

   ![Graph Window]

   **Note:** Do not reconfigure the set values for all the parameters
iv. Proceed to open the next window by clicking “next”

v. Input the furnace temperature and magnet excitation current in the “Title of measurement” box (e.g. 180deg 0.4A) for easy identification

vi. Adjust the magnet excitation current to 0.4 A as registered on the ammeter

vii. Click “start measurement”. It will take about 2 minutes to complete the scanning by the step motor. The graph will be plotted on the top left screen

viii. Upon completion of the plot, click “at start position” to reset to starting position

ix. Click “end series of measurement”

x. The graph plotted will be displayed in full screen view as shown below.
I. ANALYSIS

i. Go to “Analysis” menu, click “smooth” and check “into new measurement” box

ii. Next click “smooth”

The resulting graph will be as shown.

![Graph showing a smooth curve]

iii. At the main menu, click the “+” icon and drag an area that covers the first peak

iv. Go to “Analysis” menu, click “show extreme” and check “visualize result”

The 1st peak will be labelled on the plot as shown below.

![Graph with labelled peaks]

v. Repeat steps iii & iv for the second peak
vi. Note the x position (in mm) of both peaks and calculate the
distance between them

vii. Multiply the measured distance by \( \frac{1}{3} \) to obtain the actual
ionization current peaks \( 2u \).

viii. Save the graph using a filename for easy identification
such as “180deg 0.4A_date”

ix. Repeat the experiment with magnet excitation current of
0.5 A, 0.6 A, 0.7 A, 0.8 A, 0.9 A

5. For each magnet excitation current \( I \), determine
   i) the magnet induction \( B \) from the calibration curve given in the
      Appendix.
   ii) \( \frac{\partial B}{\partial z} \) using eq.(5) with \( a = 2.5 \) mm
   iii) separation of ionization current peaks \( 2u \) from the plot, from
       which obtain \( u_e \).
   iv) \( 3u_e - \frac{c}{u_e} \) using eq (16) with \( D = 0.86 \) mm and \( p = 0.36 \) mm.

6. Tabulate \( i, B, \frac{\partial B}{\partial z}, 2u, u_e, 3u_e - \frac{c}{u_e} \)

7. Plot \( \frac{\partial B}{\partial z} (T/m) \) vs \( 3u_e - \frac{c}{u_e} \) (mm) and determine its slope.

8. Using eq (17) calculate the Bohr magneton \( \mu_B \) with \( I = 0.455 \) m,
   \( L = 7 \) cm, \( T = 273+160 = 433k \) (for furnace temperature = 180 °C) and
   slope obtained above.