Objectives

- Demonstrate the dispersion of light through a prism and the principle of a prism spectrometer.
- Determine the index of refraction of a glass prism for various wavelengths.
- Determine the experimental values for the Cauchy’s constants for the glass prism.
- Determine an experimental value for the wavelength of sodium D line from the calibrated prism.

Equipment List

- Prism spectrometer
- Mercury discharge lamp
- Sodium discharge lamp

Theory

Figure 1: Illustration of refraction of light rays at a plane surface.

A light ray incident on a plane surface that forms the boundary between two transparent media can be reflected at the surface or transmitted into the second medium. In general, light rays incident on a plane interface will be partially reflected and partially transmitted. The ray that is transmitted does not continue in the same direction. Instead, it undergoes a change in direction – refraction. The angle of incidence is $\theta_1$ and the angle of refraction is $\theta_2$. The change in direction of the light ray is caused by the fact that the speed of light is different for different media.
The speed of light in a vacuum is \( c \) and it represents the maximum possible speed of light. For any material, the speed of light is \( v \) where \( v \leq c \). A quantity called the index of refraction or \( n \) for any medium is defined by \( n = c/v \). Note that since \( v \leq c \), then the only allowed values of \( n \) are \( n \geq 1 \).

The relationship between the angle of incidence \( \theta_1 \) and the refracted angle \( \theta_2 \) is an expression involving the indices of refraction of the two media \( n_1 \) and \( n_2 \). The relationship is known as Snell’s law and it is

\[
\frac{n_1 \sin \theta_1}{n_2 \sin \theta_2} = 1
\]

When \( n_1 > n_2 \), equation (1) implies that \( \theta_1 < \theta_2 \). This states that a ray going from a medium of a given index of refraction to one of a smaller index of refraction is bent away from the normal. In \( n_1 < n_2 \), then it must be true that \( \theta_1 > \theta_2 \) and thus a ray going into a medium of larger index of refraction is bent toward the normal.

In general, the index of refraction is not the same for all light wavelengths (colours). This implies that, for a given incident light, the refraction angle will on the wavelength of the incident light. The phenomenon whereby the index of refraction of a material measurably depends on the wavelength of the incident light is called dispersion. Empirically, the relationship between the index of refraction and the wavelength of light is known as Cauchy’s formula

\[
n = A + \frac{B}{\lambda^2}
\]

where \( A \) and \( B \) are called Cauchy’s constants.

Figure 2: Geometric relations for refracted rays within a prism.

Figure 3: Geometric relations when light passes through a prism symmetrically.

An optical optical prism is a transparent, optically dispersive material which is typically cut in the shape of the triangular wedge. Consider a light ray obliquely incident on left hand side of a triangular prism as shown in Figure 2. This figures shows a typical path for the light ray transiting this material. The angles \( \theta_1 \), \( \theta_2 \), \( \theta'_1 \) and \( \theta'_2 \) are, respectively, the incident angle from medium 1 to medium 2, the refraction angle within medium 2, the incident angle
from medium 2 to medium 1, and the refraction angle out into medium 1. There are related according to Snell’s law as follows:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{and} \quad n_2 \sin \theta'_1 = n_1 \sin \theta'_2 \]  

(3)

The light exits the prism through right hand side at an angle \( \theta'_2 \) with respect to the normal to side \( B \). If we draw a line along the direction of the incident ray through the prism, it will intersect side \( B \) at an angle of \( \omega \) with respect to the normal to that surface. The deviation angle is defined as the angle between the initial direction of the light ray and its final direction. Inspection on Figure 2 shows that this angle is given by

\[ \delta = \theta'_2 + \omega \]  

(4)

It can be further shown that

\[ \delta = \theta_1 + \theta'_2 - \alpha \]  

(5)

where \( \alpha \) is the angle formed by the leading edge of the prism.

If the angle of incidence \( \theta_1 \) is varied from some arbitrary starting value, either by moving the light source or rotating the prism, the deviation angle \( \delta \) will become smaller until it reaches a minimum value, after which continued adjustment of \( \theta_1 \) will cause the deviation angle to increase again. This special, smallest, value of \( \delta \) is called the angle of minimum deviation which is denoted by \( \delta_m \). The angle of minimum deviation is important because, for this particular angle, a relatively simple relationship between \( \delta_m \) and the index of refraction can be developed.

One can use differential calculus to show that the condition for \( \delta \) to be minimum is

\[ \theta_1 = \theta'_2 \quad \text{and} \quad \theta_2 = \theta'_1 \]  

(6)

A light ray incident on the prism at an angle \( \theta_1 \) such that the above condition is satisfied will pass through the prism symmetrically as in Figure 3. Further geometric manipulations reveal that when the minimum deviation conditions are satisfied, \( \theta_1 \) and \( \theta_2 \) obey rather simple conditions:

\[ \theta_2 = \frac{\alpha}{2} \]  

(7)

and

\[ \theta_1 = \theta_2 + \frac{\delta_m}{2} = \frac{\alpha + \delta_m}{2} \]  

(8)

Taking the index of refraction of air to be \( n_1 = 1 \) and inserting the above relations for \( \theta_1 \) and \( \theta_2 \) into Snell’s law gives a simple equation for the index of refraction of the prism, \( n_2 \), in terms of the angle of minimum deviation:

\[ n_2 = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin \left[ \frac{1}{2} (\alpha + \delta_m) \right]}{\sin (\alpha/2)} \]  

(9)

In a dispersive material, the index of refraction depends on the wavelength of the incident light \( \lambda \). By measuring the angle of minimum deviation for different light wavelengths, we can determine the wavelength dependence of the index of refraction by using the equation

\[ n(\lambda) = \frac{\sin \theta_1}{\sin \theta_2} = \frac{\sin \left[ \frac{1}{2} (\alpha + \delta_m(\lambda)) \right]}{\sin (\alpha/2)} \]  

(10)
In this experiment, we will use a prism spectrometer to measure the deviations of light for various wavelength. The spectrometer is an instrument for studying optical spectra. A schematic diagram of a prism spectrometer is shown in Figure 4. It consists of a collimator, a telescope, a prism and a circular spectrometer table. The collimator holds an aperture at one end that limits the light coming from the source to a narrow rectangular slit. A lens at the other end focuses the image of the slit onto the face of the prism. The telescope magnifies the light exiting the prism and focuses it onto the eyepiece. The prism, of course, disperses the incident light into its constituent wavelengths. The vernier scale allows the angles at which the collimator and telescope are located to be read off.
4 Laboratory Work

In this experiment, a calibration curve for the prism, a graph of wavelength versus index of refraction, will first be constructed using mercury spectrum. The wavelength of the sodium D line will then be determined experimentally from the calibration curve.

1. Remove the prism from the spectrometer.

2. Telescope Setup.
   - While looking through the telescope, slide the eyepiece in and out until the cross-hairs come into sharp focus.
   - Loosen the graticule lock ring, and rotate the graticule until one of the cross-hairs is vertical. Retighten the lock ring and then refocus if necessary.
   - Position the spectrometer so that the telescope can be pointing at some distant object (e.g. building outside the window).
   - View the distant object through the telescope and turn the focus knob until the image is sharp. The telescope is now focussed for parallel light rays.

3. Position the mercury discharge lamp close to the slit at the end of the collimator. Turn on the mercury discharge lamp.

![Figure 5: Align the Telescope directly opposite the Collimator.](image)

   - Check that the collimator slit is partially open (use the slit width adjust screw).
   - Align the telescope directly opposite the collimator as shown in Figure 5.
   - Looking through the telescope, adjust the focus of the collimator and, if necessary, the rotation of the telescope until the slit comes into sharp focus. DO NOT change the focus of the telescope.
   - Adjust the slit width until the image of the slit as seen through the telescope is sharply focused on the cross-wires. If the slit is not vertical, loosen the slit lock ring, realign the slit, and retighten the lock ring. The collimator is then set to produce parallel light from the slit.
5. **Spectrometer Table Setup.**

- Place the prism at the center of the spectrometer table so that one of the refracting faces (say $XY$) is approximately perpendicular to a line joining two leveling screws 1 and 2 of the spectrometer table (see Figure 6).
- Adjust the height of the spectrometer table until the collimator is centered on the vertical dimension of the prism.
- Rotate the spectrometer table and telescope until a refracted slit image is observed. If the image is not in the center of the field of view, the refracting edge of the prism is not parallel to the axis of rotation of the telescope and the spectrometer table will need to be leveled.
- Rotate the spectrometer table and telescope so that a reflected slit image can be observed from face $XY$. Adjust leveling screw 2 until the reflected slit image is in the center of the field of view.
- Rotate the spectrometer (but not the telescope) so that a reflected slit image can be observed from face $YZ$. Adjust leveling screw 3 until the reflected slit image is in the center of the field of view.

**Note:** To read the angle, first find where the zero point of the vernier scale aligns with the degree plate and record the value. In Figure 7, the zero point on the vernier scale is between the $155^\circ$ and $155^\circ30'$ marks on the degree plate, so the recorded value is $155^\circ$. Now find the line on the vernier scale that aligns most closely with any line on the degree scale. In the figure, this is the line corresponding to a measurement of 15 minutes of arc. Add this value to the reading recorded above to get the correct measurement, i.e., $155^\circ + 15' = 155^\circ15'$. 

![Figure 6: Leveling the spectrometer table.](image)

![Figure 7: Vernier scales.](image)
6. Measurement of the Prism Angle $\alpha$.

- Rotate the spectrometer table until the refracting edge of the prism is approximately pointing towards the collimator as in Figure 8. Light from the collimator will be reflected from both sides of the prism ($ZX$ and $ZY$).
- Rotate the telescope to one side to receive the light which is reflected from the face $ZX$ of the prism. Carefully position the telescope until the slit image is centered on the cross-wires. Record the angular position of the telescope as $\theta_1$ in Data Table 1.
- Rotate the telescope around to receive light reflected off the other face $ZY$ of the prism and record the angular position of the telescope as $\theta_2$ in Data Table 1.
- Rotate the spectrometer table slightly by a few degrees. Repeat the above steps FOUR more times to have a total of FIVE trials in estimating the prism angle $\alpha$. Record all your results in Data Table 1.

7. Calibration of the Prism Spectrometer.

- Remove the prism from the spectrometer and align the telescope so that the slit image is on the cross-wires as seen through the telescope.
- Record the angular position of the telescope as $\theta_0$ in Data Table 2. This will be the reference angular position for any measurements later.
- Replace the prism on the spectrometer table so that it is oriented as in Figure 9.
- Rotate the telescope so that the image of the spectrum is on the cross-wires as seen through the telescope.
- Gently rotate the prism back and forth. As you attempt to make the deviation angle smaller, the succession views of the spectrum should appear to migrate in one direction until a point at which it reverses direction and migrate back to where it came from (see Figure 10). Position the prism so that it is roughly at the reversal point.
Figure 10: Successive views as the turntable is rotated through the angle of minimum deviation.

- Position the telescope so that you can view the spectrum when the prism is oriented at the reversal point. Make fine adjustments to the prism so as to precisely pin down the reversal point.
- Locate and record the angular positions of the telescope for at least FIVE different wavelengths for the mercury spectrum. Record your respective data in Data Table 2.
- Repeat the above steps so that THREE sets of data are obtained for each wavelength. The other sets of data could be obtained by orientating the prism at different locations as shown in Figure 9.

  **Hint:** When measuring very closely spaced lines, like the double yellow lines, make the slit very narrow. When measuring dim lines, make the slit wider.

8. Place the solidum discharge lamp in front of the collimator slit. Repeat the above procedures FIVE times to determine the angular position of the telescope when the sodium D line (which is yellow and extremely bright) passes through the prism symmetrically. Record your data in Data Table 3.