The Photoelectric Effect

In the late 1800s, experiments showed that light incident on certain metal surfaces caused electrons to be emitted from the surfaces. This is called the photoelectric effect.

In a typical apparatus for studying the photoelectric effect, light of frequency $\nu$ falls on a metal plate and kicks out electrons. A suitable potential difference $V$ between the metal plate and collector cup sweeps up these photoelectrons, displaying them as a photoelectric current which can be measured by an ammeter.

The stopping potential $V_0$ is the potential difference required to stop the fastest photoelectrons, thus bringing the photoelectric current to zero. It was found that $V_0$, and therefore, the kinetic energy of the most energetic photoelectrons ($K_m = eV_0$), do not depend on the intensity of the incident light.

Furthermore, it was found that there exists a cutoff frequency $\nu_0$. For light with frequency below this value the photoelectric effect simply does not happen.

Einstein’s theory: “collision” between an incident photon and an electron within the metal.
An apparatus used to study the photoelectric effect. The incident light falls on plate $P$, ejecting photoelectrons, which are collected by collector cup $C$. 
A plot (not to scale) of data taken in a photoelectric experiment. The intensity of the incident light is twice as great for curve $b$ as for curve $a$. The wavelength of the incident light remains the same for each curve.
The stopping potential for sodium as a function of the frequency of the incident light.
The Intensity Problem

In wave theory, when you increase the intensity of a beam of light, you increase the magnitude of the oscillating electric field vector, $\vec{E}$. The force that the incident beam exerts on an electron is $e\vec{E}$. You should then expect that the more intense the light, the more energetic would be the ejected photoelectrons. However, experiment results tested over an intensity range of $\sim 10^7$ showed that $V_0$ (and thus $K_m$) does not depend on the light intensity.

Using the photon picture, the “intensity problem” is no problem. If we double the light intensity, we simply double the number of photons but we do not change the energy of the individual photons. Thus $K_m$, the maximum kinetic energy that an electron can pick up from a photon during a collision, remains unchanged.
The Frequency Problem

According to wave theory, the photoelectric field should occur at any frequency of the incident light, provided only that the light is intense enough. However, experiment shows that there is a characteristic cutoff frequency below which there is no photoelectric effect, no matter how intense the light.

If we think in terms of photons, the “frequency problem” is no problem. The conduction electrons are held within the metal target by a potential barrier at the metal surface. Thus, to eject a photoelectron, you must give it a certain minimum energy $\phi$, called the work function of the material. If the photon energy exceeds the work function (that is, if $h\nu > \phi$), the photoelectric effect can occur.
The Photoelectric Effect

Einstein’s Theory: not only explains the photoelectric effect, it also confirmed the particle nature of light.

**Photon**

\[ E = h\nu = \hbar \omega \]
\[ \vec{p} = \frac{h\nu}{c} = \frac{h}{\lambda} = \hbar \vec{k} \]

Left hand sides: quantities describing a particle
Right hand sides: quantities describing a wave

**Planck’s constant**

\[ h\nu = \phi + K_m = \phi + eV_0 \]
\[ V_0 = \frac{h}{e}\nu + \frac{\phi}{e} \]

\( h \) can be obtained from the \( V_0 \) vs. \( \nu \) graph,

\[ h = 6.626 \times 10^{-34} \text{J} \cdot \text{s} \]
Compton's Scattering

The experiment done by Authur Holly Compton in 1923 proved to be a great “conviner” of the reality of photons because it introduced the photon momentum, as well as the photon energy, into the experimental situation.

The experiment

A beam of x rays of wavelength $\lambda$ falls on a graphite target. Compton measured, as a function of wavelength, the intensity of the x rays scattered from the target in several selected directions. It was found that even though the incident beam contains only a single wavelength, the scattered x rays have intensity peaks at two wavelengths. One peak corresponds to the incident wavelength $\lambda$; the other has a wavelength $\lambda'$ that is longer than $\lambda$ by an amount called the Compton shift.
An apparatus used to study the Compton effect. A beam of x rays falls on a graphite target T. The x rays scattered from the target are observed at various angles to the incident direction. The detector measures both the intensity and the wavelength of these scattered x rays.
Compton’s results, for four values of the scattering angle. The Compton shift increases as the scattering angle increases.
Compton's Scattering

Classical picture:

The incident wave, with frequency $\nu$, causes the electrons in the target to oscillate at that same frequency. These oscillating electrons, like charges surging back and forth in a small transmitting antenna, radiate at this same frequency. Thus the scattered beam should have only the same frequency as the incident beam. The scattered peak of wavelength $\lambda'$ cannot be understood at all if you think of the incident x-ray beam as a wave.

Compton’s view:

Compton viewed the incident beam as a stream of photons of energy $E$ ($= h\nu$) and momentum $p$ ($= h/\lambda$) and assumed that some of these photons made billiard ball-like collisions with individual free electrons in the target. Since the electron picks up some kinetic energy in such an encounter, the scattered photon must have a lower energy $E'$ than the incident photon. It will therefore have a lower frequency $\nu'$ and, correspondingly, a longer wavelength $\lambda'$. This accounts quantitatively for the Compton shift.
Quantitative Analysis

Refer to figure on next page

Before collision

Photon

\[ E_p = h\nu, \quad \vec{p}_p = \hbar \vec{k} \]

Electron

\[ E_e = mc^2, \quad \vec{p}_e = 0 \]

After collision

Photon

\[ E'_p = h\nu', \quad \vec{p}'_p = \hbar \vec{k}' \]

Electron

\[ E_e = \frac{mc^2}{\sqrt{1 - \beta^2}}, \quad \vec{p}'_e = \frac{m\nu}{\sqrt{1 - \beta^2}} \]

where

\[ \beta = \frac{v}{c} \]
Quantitative Analysis

Energy conservation

\[ h\nu + mc^2 = h\nu' + \frac{mc^2}{\sqrt{1 - \beta^2}} \]  \hspace{1cm} (1)

Momentum conservation

\[ \hbar k = \hbar k' \cos \phi + \frac{mv \cos \theta}{\sqrt{1 - \beta^2}} \]  \hspace{1cm} (2)

\[ 0 = \hbar k' \sin \phi - \frac{mv \sin \theta}{\sqrt{1 - \beta^2}} \]  \hspace{1cm} (3)

We want to know the information about the photon after scattering, \( \lambda' \) (or \( \nu' \)), \( \phi \). Since we are not interested in the state of the electron (\( \nu \) and \( \theta \)), we can eliminate \( \nu \) and \( \theta \) from the three equations.
Quantitative Analysis

To eliminate $\theta$, we rewrite Eq.(2) and Eq.(3) as

$$\frac{mv \cos \theta}{\sqrt{1 - \beta^2}} = \hbar k - \hbar k' \cos \phi \quad (4)$$

$$\frac{mv \sin \theta}{\sqrt{1 - \beta^2}} = \hbar k' \sin \phi \quad (5)$$

Square both sides of Eqs. (4) and (5) and add the two equations, we get (using $p = \hbar k = h/\lambda$)

$$\frac{m^2 v^2}{1 - \beta^2} = \left(\frac{\hbar}{\lambda}\right)^2 + \left(\frac{\hbar}{\lambda'}\right)^2 - 2 \frac{\hbar^2}{\lambda \lambda'} \cos \phi \quad (6)$$

Next, we square both sides of Eq.(1) and then eliminate $v$ using Eq.(1) and Eq.(5). After some reorganization, we have

$$\Delta \lambda = \lambda' - \lambda = \frac{\hbar}{mc} (1 - \cos \phi)$$