Development of Quantum Mechanics

Reference

- PC1133 Lectures 8 – 12
- Serway: Physics for Scientists and Engineers with Modern Physics
Development of Quantum Mechanics

Physics in late 19th century

Classical physics was nearly perfect.

- Mechanical motion — Newton mechanics
- Electricity & Magnetism — Maxwell’s theory
- Optics (EM wave) — Maxwell’s theory
- Thermal phenomena — Thermodynamics & Statistical physics

New Challenges

- Blackbody radiation
- Photoelectric effect
- Atomic spectrum
- Specific heat of solids at low temperature
- . . . . .

⇒ Quantum Mechanics
Wave Property of Light

The wave property of light was established in the 17th century. This was demonstrated by the Young’s double slit experiment.

Experiment Setup

*see diagram on next page*

Intensity Analysis

Difference in paths

\[ \delta = d \sin \theta \]

Phase difference

\[ \phi = \frac{2\pi}{\lambda} \delta = \frac{2\pi}{\lambda} d \sin \theta \]

If

\[ E_1 = E_0 \sin(\omega t) \]

then

\[ E_2 = E_0 \sin(\omega t + \phi) \]
Geometric construction for describing Young’s double-slit experiment.
Wave Property of Light

Amplitude at $P$

$$E_p = E_1 + E_2 = E_0 \left[ \sin(\omega t) + \sin(\omega t + \phi) \right]$$

Using

$$\sin \alpha + \sin \beta = 2 \sin \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)$$

$$E_p = 2E_0 \cos \left( \frac{\phi}{2} \right) \sin \left( \omega t + \frac{\phi}{2} \right)$$

Intensity at $P$

$$I \propto E_p^2 = 4E_0^2 \cos^2 \left( \frac{\phi}{2} \right) \sin^2 \left( \omega t + \frac{\phi}{2} \right)$$

Time average of $\sin^2(\omega t + \phi/2)$ is 1/2.

$$I_{av} = I_0 \cos^2 \left( \frac{\phi}{2} \right)$$
Wave Property of Light

\[ I_{av} = I_0 \cos^2 \left( \frac{\pi d}{\lambda} \sin \theta \right) = I_0 \cos^2 \left( \frac{\pi d}{\lambda L} y \right) \]

Maximum intensity at

\[ d \sin \theta = n\lambda, \quad (n : \text{integer}) \]

or

\[ \frac{\pi d}{\lambda L} y = n\pi, \quad y_{max} = \frac{n\lambda L}{d} \]

Minimum intensity at

\[ d \sin \theta = \left( n + \frac{1}{2} \right) \lambda \]

or

\[ \frac{\pi d}{\lambda L} y = \left( n + \frac{1}{2} \right) \pi, \quad y_{max} = \left( n + \frac{1}{2} \right) \frac{\lambda L}{d} \]

\[ \implies \text{Light is a wave!} \]
Intensity distribution versus $d \sin \theta$ or the double-slit pattern when the screen is far from the two slits.
Blackbody Radiation

Observation

- Radiation, reflection, absorption
- Blackbody
- Radiation spectrum
  - Shape and peak position
  - $\lambda_{\text{max}}T = \text{const.}$
  - Depends on $T$ only

Classical theory (Rayleigh-Jeans)

$$I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}$$
Intensity of blackbody radiation versus wavelength at three temperatures. Note that the amount of radiation emitted (the area under a curve) increases with increasing temperature.
Comparison of the experimental results with the curve predicted by the Rayleigh-Jeans classical model for the distribution of blackbody radiation.
Planck’s Theory

Assumptions

1. Energy is quantized

\[ E_n = n h \nu \]

2. Light is emitted or absorbed in discrete packets called photons

\[ E = h \nu \]

\( \nu \) is the frequency.

\[ I(\lambda, T) = \frac{2\pi h c^2}{\lambda^5 \left( e^{hc/\lambda k_B T} - 1 \right)} \]

The concept of “quantum” was introduced for the first time: energy is quantized!