Objectives

• Demonstrate the specific knowledge gained by repeated physical measurements for the mass, length, inner and outer diameters of a hollow cylinder.
• Apply the statistical concepts of mean, standard deviation from the mean and standard error to these measurements.
• Demonstrate propagation of uncertainties by determining the uncertainty in the volume and density calculated from the measured quantities.
• Demonstrate the method of linear least squares fits by determining a value of the mathematical constant $\pi$ from measured diameters and circumferences of circles.

Equipment List

• Meter stick
• Vernier caliper
• Micrometer caliper
• Digital balance
• Hollow metal cylinder
• Circles set

Theory

§3.1 Least Count and Significant Figures

In general, there are exact numbers and measured numbers. Factors such as the 100 used in calculating percentage and the 2 in $2\pi r$ are exact numbers. Measured numbers, as the name implies, are those obtained from measurement instruments and generally involve some uncertainties.

In reporting experimentally measured values, it is important to read one’s instruments correctly. The degree of uncertainty of a number read from a measurement instrument depends on the quality of instrument and the fitness of its measuring scale. When one is reading the value from a calibrated scale, only a certain number of figures can properly be obtained or
read. That is, only a certain number of figures are *significant*. This depends on the *least count* of the instrument scale, which is the smallest subdivision on the measurement scale. This is the unit of the smallest reading that can be made without estimating. For example, the least count of a meter stick is usually the millimeter (mm). We commonly say “the instrument is calibrated in centimeters (numbered major divisions) with a millimeter least count” (see Figure 1).

The *significant figures* of a measured value include all the numbers that can be read directly from the instrument scale plus one estimated number – the fractional part of the least count smallest division. For example, the length of the rod in Figure 1 (as measured from the zero end) is 2.64 cm. The rod’s length is known to be between 2.6 cm and 2.7 cm. The estimated fraction is taken to be 4/10 of the least count (1 mm), so the estimated figure is 4, giving 2.64 cm with three significant figures.

For an instrument with a digital display, the estimated fraction is always taken to be one half of the smallest division that appears on the display. For example, if the time interval indicated by a digital stopwatch is 1.43 s, the estimated fraction is taken to be 0.005 s of the least count 0.01 s. The time interval is known to be between 1.425 s and 1.435 s. The reported value is then 1.430 s with four significant figures.

Thus, measured values contain inherent uncertainty because of the estimated figure. However, the greater the number of significant figures, the greater the reliability of the measurement the number represents. For example, the length of an object may be read as 2.54 cm (three significant figures) on one instrument scale and as 2.5404 cm (five significant figures) on another. The latter reading from an instrument with a finer scale gives more information and reliability.

§3.2 The Vernier Caliper

In 1631, a Frenchman, Pierre Vernier, devised a way to improve the precision of length measurements. The vernier caliper (Figure 2), commonly called a vernier, consists of a rule with a main engraved scale and a moveable jaw with an engraved vernier scale. The span of the lower jaw is used to measure length and is particularly convenient for measuring the diameter of a cylindrical object. The span of the upper jaw is used to measure distances between two surfaces (such as, the inside diameter of a hollow cylindrical object).
Figure 2: A vernier caliper.

The main scale is calibrated in centimeters with a millimeter least count and moveable vernier scale has 10 divisions that cover 9 divisions on the main scale. The least count of the vernier, in this case, is \( \frac{1\text{ mm}}{10} = 0.1\text{ mm} \). When making a measurement with a meter stick, it is necessary to estimate the fractional part of the smallest division (the tenth of a millimeter). The function of the vernier scale is to assist in the accurate reading of the fractional part of the scale division, thus increasing the precision.

Figure 3: An example of reading the vernier scale on a caliper.

The leftmost mark on the vernier scale is the zero mark. The zero mark is often unlabeled. A measurement is made by closing the jaws on the object to be measured and reading where the zero mark on the vernier scale falls on the main scale (see Figure 3).

The first two significant figures are read directly from the main scale in Figure 3. The vernier zero mark in past the 2-mm line after the 1-cm major division mark, so we have a
reading of 1.2 cm for both (a) and (b). The next significant figure is the fractional part of
the smallest subdivision on the main scale. This is obtained by referring to the vernier scale
markings below the main scale.

If a vernier mark coincides with a mark on the main scale, then the vernier mark number
is the fractional part of the main-scale division (see Figure 3a). In the figure, this is
the third mark to the right of the vernier zero, so the third significant figure is 3 (0.03 cm). Finally, since the 0.03-cm reading is known exactly, a zero is added as the estimated figure, for
a reading of 1.230 cm or 12.30 mm. Note how the vernier scale gives more significant figures
or extends the precision.

However, a mark on the vernier scale may not always line up exactly with one on the main
scale (Figure 3b). In this case, there is more uncertainty in the 0.001-cm or 0.01-mm figure
and we say there is a change of “phase” between two successive vernier markings.

Notice how in Figure 3b the second vernier mark after the zero is to the right of the closest
main-scale mark and the third vernier mark is to the left of the next main-scale mark. Hence,
the marks change “phase” between the 2 and 3 marks which means the reading is between
1.22 cm and 1.23 cm. Most vernier scales are not fine enough for us to make an estimate of
the estimated figure, so a suggested method is take the middle of the range. Thus we would
put a 5 in the thousandth-of-a-centimeter digit, for a reading of 1.225 cm.

**Zeroing**

Before making a measurement, one should check the zero of the vernier caliper with the jaws
completely closed. It is possible that through misuse the caliper is no longer zeroed and thus
gives erroneous readings (systematic uncertainty). If this is the case, a zero correction must
be made for each reading.

Figure 4: The zero of the vernier caliper is checked with the jaws closed. (a) Zero error. (b)
Positive error, +0.05 cm.
In zeroing, if the vernier zero lies to the right of the main-scale zero, measurements will be too large and the error is said to be positive. In this case, the zero correction is made by subtracting the zero reading from the measurement reading. For example, the “zero” reading in Figure 4 is +0.05 cm and this amount must be subtracted from each measurement reading for more accurate results.

Similarly, if the error is negative, or the vernier zero lies to the left of the main-scale zero, measurements will be too small and the zero correction must be added to the measurement readings.

Summarizing these corrections in equation form,

$$\text{Corrected reading} = \text{actual reading} - \text{zero reading}$$

For example, for a positive error of +0.05 cm as in Figure 4,

$$\text{Corrected reading} = \text{actual reading} - 0.05 \text{ cm}$$

If there is a negative correction of −0.05 cm, then

$$\text{Corrected reading} = \text{actual reading} - (-0.05) \text{ cm}$$

$$= \text{actual reading} + 0.05 \text{ cm}$$

§3.3 The Micrometer Caliper

The micrometer caliper (see Figure 5) provides for accurate measurements of small lengths and is particularly convenient in measuring the diameters of thin wires and the thickness of thin sheets. It consists of a movable spindle (jaw) that is advanced toward another, parallel-faced jaw (called an anvil) by rotating the thimble. The thimble rotates over an engraved sleeve mounted on a solid frame.

![Figure 5: A micrometer caliper.](image)

Most micrometers are equipped with a ratchet (ratchet handle is to the far right in the figure) that allows slippage of the screw mechanism when a small and constant force is exerted.
on the jaw. This permits the jaw to be tightened on an object with the same amount of force each time. Care should be taken not to force the screw (particularly if the micrometer does not have a ratchet mechanism), so as not to damage the measured object and/or the micrometer.

The axial main scale on the sleeve is calibrated in millimeters and the thimble is calibrated in 0.01 mm (hundredths of a millimeter). The movement mechanism of the micrometer is a carefully machined screw with a pitch of 0.5 mm. The pitch of a screw, or the distance between screw threads, is the lateral linear distance the screw moves when turned through one rotation.

The axial line on the sleeve main scale serves as a reading line. Since the pitch of the screw is 0.5 mm and there are 50 divisions on the thimble, when the thimble is turned through one of its divisions, the thimble moves (and the jaws open or close) 1/50 of 0.5 mm, or 0.01 mm.

One complete rotating of the thimble (50 divisions) moves it through 0.5 mm and a second rotation moves it through another 0.5 mm, for a total of 1.0 mm, or one scale division along the main scale. That is, the first rotation moves the thimble from 0.00 to 0.50 mm and the second rotation moves the thimble from 0.50 through 1.00 mm.

![Figure 6: An example of a micrometer reading.](image)

Measurements are taken by noting the position of the edge of the thimble on the main scale and the position of the reading line on the thimble scale. For example, for the drawing in Figure 6, the micrometer has a reading of 5.785 mm. On the main scale is a reading of 5.000 mm plus one 0.500-mm division (scale below reading line), giving 5.500 mm. The reading on the thimble scale is 0.285 mm, where the 5 is the estimated figure; that is, the reading line is estimated to be midway between the 28 and the 29 marks.

As with all instruments, a zero check should be made and a zero correction applied to each reading if necessary as described in Section §3.2. A zero reading is made by rotating the screw until the jaw is closed or the spindle comes into contact with the anvil. The contacting surface of the spindle and anvil should be clean and free of dust.
§3.4  Density

The *density* \( \rho \) of a substance is defined as the mass \( m \) per unit volume \( V \) (i.e. \( \rho = m/V \)). Thus, the densities of substance provide comparative measures of the amounts of matter in a particular (unit) space. Note that there are two variables in density – mass and volume. Hence, densities can be affected by the masses of atoms and/or by their compactness.

As can be seen from the defining equation \( \rho = m/V \), the SI units of density are kilogram per cubic meter (kg/m\(^3\)). However, measurements are commonly made in the smaller metric units of grams per cubic centimeter (g/cm\(^3\)), which can be easily converted to standard units.

Density may be determined experimentally by measuring the mass and volume of a sample of a substance and calculating the ratio \( m/V \). The volume of regularly shaped objects may be calculated from length measurements. For example, the volume of a cube is given by \( V = \text{length} \times \text{width} \times \text{height} \).

The physical property of density can be used to identify substances in some cases. If a substance is not pure or is not homogenous (that is, its mass is not evenly distributed), an average density is obtained, which is generally different from that of a pure or homogenous substance.
Laboratory Work

Part A: Density Determination

In this part of the experiment, you will experimentally determine the density of a given hollow cylinder. The process involves measuring directly the mass, length, inner and outer diameters, and volume of the hollow cylinder, and calculating the density from these directly measured quantities.

A-1. List the least count and the estimated fraction of the least count for each of the measuring instruments in Data Table 1.

A-2. Using a digital balance, determine the mass of the hollow cylinder. Make TEN independent measurements and record them as \( m \) in Data Table 2. The balance reading should be zero with nothing on the balance pan. Otherwise, zero the digital balance by pressing the ‘ZERO’ (or ‘TARE’) button before any measurement is taken.

A-3. Place the meter stick along the length of the hollow cylinder. DO NOT attempt to line up either edge of the hollow cylinder with one end of the meter stick or with any certain mark on the meter stick.

A-4. Read the scale on the meter stick that is aligned with one end of the hollow cylinder and record that measurement as \( X_1 \) in Data Table 2. Read the scale that is aligned at the other end of the hollow cylinder and record it in Data Table 2 as \( X_2 \).

A-5. Repeat steps A-3 and A-4 nine more times for a total of TEN measurements of the length of the hollow cylinder. Make the measurements at different places along the meter stick in order to sample the variation in length of the hollow cylinder.

A-6. Close the jaws of the vernier caliper. Make a reading of the zero correction for the vernier caliper and record it in Data Table 2. Close the jaws of the micrometer caliper. Make a reading of the zero correction for the micrometer caliper and record it in Data Table 2.

Note: Record the zero correction as positive if the vernier zero is to the right of the main-scale zero. Otherwise, record it as negative if the vernier zero is to the left of the main-scale zero. The exact measurement is obtained by subtracting the zero correction from the actual reading.

A-7. Using the vernier caliper, measure the inner diameter of the hollow cylinder and record it as \( d_1 \) in Data Table 2. Using the micrometer caliper, measure the outer diameter of the hollow cylinder and record it as \( d_2 \) in Data Table 2.

A-8. Repeat step A-7 nine more times for a total of TEN separate trials. Perform these measurements at different positions along the length of the hollow cylinder in order to sample the variation in inner (outer) diameter of the hollow cylinder.
Part B: \( \pi \) Estimation

In this part of the experiment, you will measure the diameters and circumferences of a set of six circles drawn. A value of the mathematical constant \( \pi \) will be estimated based on the measured quantities.

B-1. Determine the diameter of the smallest circle (labeled \#1) and record the result as \( d \) in Data Table 3.

B-2. Repeat step B-1 four more times to have a total of FIVE trials for the diameter of the circle.

B-3. Determine the circumference of the smallest circle (labeled \#1) and record it as \( C \) in Data Table 3.

B-4. Repeat step B-3 four more times to have a total of FIVE trials for the circumference of the circle.

B-5. Repeat steps B-1 through B-4 for the rest of the circles (labeled 2–6 in the sequence of increasing size).