**Diffraction of a Circular Aperture**

Diffraction can be understood by considering the wave nature of light. Huygen's principle, illustrated in the image below, states that each point on a propagating wavefront is an emitter of secondary wavelets. The combined locus of these expanding wavelets forms the propagating wave. Interference between the secondary wavelets gives rise to a fringe pattern that rapidly decreases in intensity with increasing angle from the initial direction of propagation. Huygen's principle nicely describes diffraction, but rigorous explanation demands a detailed study of wave theory.

![Huygen's principle](image)

Diffraction effects are traditionally classified into either Fresnel or Fraunhofer types. Fresnel diffraction is primarily concerned with what happens to light in the immediate neighborhood of a diffracting object or aperture. It is thus only of concern when the illumination source is close to this aperture or object. Consequently, Fresnel diffraction is rarely important in most optical setups.

Fraunhofer diffraction, however, is often very important. This is the light-spreading effect of an aperture when the aperture (or object) is illuminated with an infinite source (plane-wave illumination) and the light is sensed at an infinite distance (far-field) from this aperture.

It is Fraunhofer diffraction that determines the limiting performance of optical systems. Diffraction, poses a fundamental limitation on any optical system. Diffraction is always present, although its effects may be masked if the system has significant aberrations. When an optical system is essentially free from aberrations, its performance is limited solely by diffraction, and it is referred to as diffraction limited.
Fraunhofer Diffraction by a Circular Aperture

Each ring is separated by a circle of zero intensity. The irradiance distribution in this pattern can be described by

\[ I_x = I_0 \left[ \frac{2J_1(x)}{x} \right]^2 \]

where \( I_0 \) = peak irradiance in the image

\[ J_1(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-2}}{(2n)!} \frac{2^{2n-1}}{2^{2n-1}} \]

\( J_1(x) \) is a Bessel function of the first kind of order unity, and

\[ x = \frac{\pi D}{\lambda} \sin \theta \]

Where \( x \) is the position on the image plane, \( \lambda \) is the wavelength, \( D \) is the aperture diameter, and \( \theta \) is the angular radius from pattern maximum. This useful formula shows the far-field irradiance distribution from a uniformly illuminated circular aperture of diameter, \( D \).

Note: For comparison, the Fraunhoffer diffraction of a uniformly illuminated slit aperture is described by

\[ I_x = I_0 \left[ \frac{\sin x}{x} \right]^2 \]

Where \( I_0 \) is the peak irradiance of the image, and
where $\lambda$ is the wavelength, $w$ is the slit width, and $\theta$ is the angular radius from pattern maximum.

**Energy Distribution Table**

The table below shows the major features of pure (unaberrated) Fraunhofer diffraction patterns for circular apertures. The table shows the position, relative intensity, and percentage of total pattern energy corresponding to each ring or band.

**Energy Distribution in the Diffraction Pattern of a Circular Aperture**

<table>
<thead>
<tr>
<th>Ring or Band</th>
<th>Position ($\lambda$)</th>
<th>Relative Intensity ($I/I_0$)</th>
<th>Energy in Ring (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Maximum</td>
<td>0.0</td>
<td>1.0</td>
<td>83.8</td>
</tr>
<tr>
<td>First Dark</td>
<td>$1.22\pi$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>First Bright</td>
<td>$1.64\pi$</td>
<td>0.0175</td>
<td>7.2</td>
</tr>
<tr>
<td>Second Dark</td>
<td>$2.23\pi$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Second Bright</td>
<td>$2.68\pi$</td>
<td>0.0042</td>
<td>2.8</td>
</tr>
<tr>
<td>Third Dark</td>
<td>$3.24\pi$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Third Bright</td>
<td>$3.70\pi$</td>
<td>0.0016</td>
<td>1.5</td>
</tr>
<tr>
<td>Fourth Dark</td>
<td>$4.24\pi$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Fourth Bright</td>
<td>$4.71\pi$</td>
<td>0.0008</td>
<td>1.0</td>
</tr>
<tr>
<td>Fifth Dark</td>
<td>5.24$\pi$</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>
Writing \( x = m \pi \), then
\[
\sin \theta = \frac{m \lambda}{D}
\]

For small angle \( \theta \),
\[
x \approx y \frac{m \lambda}{D}
\]

Experiment 1

1. Observe the diffraction pattern on a screen at a suitable distance. Photograph the pattern with a digital camera.

Make sure firstly that the laser, focusing microscope lens and pinhole are aligned along the optical axis. Check that the laser is focused sharply on the pinhole so that most of the laser light passes through the pinhole.

   Use laser goggle for eye protection

2. Set up the digital scanner to scan carefully across the pattern for a pinhole of 100 \( \mu \text{m} \) to capture on the computer the intensity profile of the diffraction pattern. Determine the relative intensity \( I_x/I_o \).

3. Repeat the scanning process for pinholes of 50 \( \mu \text{m} \) and 25 \( \mu \text{m} \).

4. Plot a suitable graph to obtain the best values for \( m \) for the first dark ring and second dark ring.

The pinhole is a delicate precision optical component.
It can be damaged by touching with your fingers or objects.
Do not damage the pinhole in your handling.
Experiment 2

Place a conventional large aperture after the pinhole to block the higher order rings so that only the center Airy pattern is projected on the screen. This is commonly known as spatial filtering for obtaining a smooth expanded laser beam without speckles.

Attach a short segment of your hair on the metal frame and place it close to the large aperture. Photograph the diffraction pattern and explain the pattern observed.