Mini-symposium A3
Modeling and Computation of Fractional Partial Differential Equations

Organizer:
Changpin Li
Department of Mathematics
Shanghai University
Email: fdc201026@yahoo.com or lcp@shu.edu.cn

A03-01 Invited
Mathematical analysis and fast numerical methods for space-fractional diffusion equations
Hong Wang
Shandong University, China and University of South Carolina, USA

Fractional diffusion equations provide an appropriate description of transport processes that exhibit anomalous diffusion, which can not be modeled properly by second-order diffusion equations. However, fractional differential equations raise mathematical and numerical difficulties that have not been encountered in the context of second-order differential equations.

Because of the nonlocal property of fractional differential operators, the numerical methods for fractional diffusion equations often generate dense coefficient matrices. Consequently, these methods often require computational work of $O(N^3)$ to invert per time step and memory of $O(N^2)$ for where $N$ is the number of unknowns. Mathematically, fractional differential equations exhibit mathematical properties that have fundamental differences from those of second-order differential equations.

In this talk we go over the development of faithful and efficient numerical methods for space-fractional partial differential equations, but rather by exploring the structure of the coefficient matrices. These methods have computational cost of $O(N \log^2 N)$ per time step and memory of $O(N)$, while retaining the same accuracy and approximation property of the underlying numerical methods. We will also address those mathematical issues that are characteristic for fractional differential equations and report our recent progress in this direction.

A03-02 Invited
High-order algorithms for Riesz derivative and their applications
Hengfei Ding, Changpin Li
Department of Mathematics, Shanghai University, Shanghai 200044, China

Numerical methods for fractional calculus attract increasing interests due to its wide applications in various fields such as physics, mechanics, etc. In this paper, we focus on constructing high-order algorithms for Riesz derivatives, where the convergence orders cover from the second order to the sixth order. Then we apply the established schemes to the Riesz space fractional turbulent diffusion equation. Numerical experiments are displayed which support the theoretical analysis.

A03-03 Invited
Finite Element Multigrid Method for Multiterm Time Fractional Advection Diffusion Equations
Weiping Bu1*, Yifa Tang1, Jiye Yang2
1 LSEC, ICMSEC, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China
2 School of Mathematics and Computer, Ningxia University, Yinchuan 750021, China

In this paper, a class of multiterm time fractional advection diffusion equations (MTFADEs) is considered. A fully discrete scheme of MTFADEs is developed by finite difference method in time direction and finite element method in space direction. The stability and convergence of the numerical method are discussed. Next, a V-cycle multigrid method is proposed to solve the resulting linear systems. The convergence of the multigrid is investigated. Finally, some numerical examples are given for verification of our theoretical analysis.

A03-04 Keynote
Efficient spectral-galerkin methods for separable fractional partial differential equations
Zhiping Mao and Jie Shen
Purdue University and Xiamen University

We consider spectral approximations to multi-dimensional separable fractional differential equations in space and in space-time. We develop efficient algorithms to compute the system matrices and to solve the corresponding linear system associated with the spectral-Galerkin discretizations.

A03-05 Invited
Fast Approximate Inversion of Block Triangular Toeplitz Matrix with Applications to Fractional Sub-Diffusion Equations
Xin Lu1, Hong-kai Pang2, Hai-wei Sun3*
1 University of Macau, 2 Jiangsu Normal University

A fast approximate inversion method is proposed for the block lower triangular Toeplitz with tri-diagonal blocks (BL3TB) matrix. The BL3TB matrix is approximated by a block circulant-like matrix, which can be efficiently inverted using the fast Fourier transforms. The good approximation is proved under a certain condition. As applications, the discretized matrix by a finite difference method for the fractional sub-diffusion equation is shown as a BL3TB matrix and satisfies this condition. Therefore, the proposed method can be efficiently applied to find the numerical solution of the fractional sub-diffusion equation. Numerical experiments are carried out to demonstrate the excellence performance of the proposed method.

A03-06 Invited
Tempered Fractional Sturm-Liouville Eigen-Problems
Mohsen Zayernouri *, Mark Anisworth, and George Em Karniadakis
Division of Applied Mathematics, Brown University, USA

In this talk, we introduce two classes of regular and singular Tempered Fractional Sturm-Liouville Problems of two kinds (TFSLP-I and TFSLP-II) of order $s \in (0,1)$. These tempered fractional differential operators are of tempered Riemann-Liouville and tempered Caputo type of fractional. We show the well-posedness of the boundary-value problems and that the eigenvalues of the regular tempered problems are real-valued and the corresponding eigenfunctions are orthogonal. Next, we obtain the explicit eigensolutions to the TFSLP-I and -II as non-polynomial functions, which we define as “Tempered Jacobi Poly-Fractonomials”. These eigenfunctions are orthogonal with respect to the weight function associated with the TFSLP-I and -II. Finally, we introduce these eigenfunctions as new basis (and test) functions for spectrally accurate approximation of functions and tempered-fractional differential operators. To this end, we further develop a Petrov-Galerkin spectral method for solving Tempered Fractional ODEs (TFODEs), followed by the corresponding stability and convergence analysis, which validates the achieved spectral convergence in our simulations.

A03-07 Invited
The discontinuous finite element method for fractional cable equation
Yunying Zheng1 *, Zhengang Zhao2
1 School of Mathematical sciences, Huaibei Normal University, Huaibei
2 Department of Fundamental Course, Shanghai Customs College, Shanghai

The cable equation as the one of the best models for simulating
neurodynamics can be derived from the Nernst-Planck equation which simulating the electrodiffusion of ions. Recently, some researchers find that in nerve cells when molecular diffusion is anomalous subdiffusion, it is much more effective using the fractional cable equation for simulating the dynamic behavior of neurons. And the fractional cable equation has attracted more and more attention.

This paper constructs a full discrete numerical method for solving the fractional cable equation, which in time axis trying the discontinuous finite element method and in special axis adopting the Galerkin finite element scheme. The existence and uniqueness are included, the convergence are also discussed in detail. The numerical examples are given to certify the theoretical results.

A03-08 Invited
Moving mesh finite element methods for fractional differential equations
Jingtang Ma*, Zhiquang Zhou
School of Mathematics, Southwestern University of Finance and Economics, No. 555, Liu-Tai-Da-Dao Street, Wenjiang Area, Chengdu, China

Most existent papers have focused on the fixed mesh methods for solving fractional differential equations. However since some classes of fractional differential equations may have singular or even finite-time blowup solutions, it is highly needed to develop adaptive mesh methods to solve these problems.

In this talk I will present the moving finite element methods for a class of fractional differential equations. The convergence theories of the methods are derived and numerical examples are provided to support the theoretical results. In the analysis, a fractional Ritz projection operator is introduced and the error estimation of the projection is derived under the moving mesh setting.

A03-09 Invited
Understanding partial bed-load transport: experiments and stochastic model analysis
HongGuang Sun1,*, Dong Chen2, Yong Zhang3,4, Li Chen4
1State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, College of Mechanics and Materials, Hohai University, Nanjing, China
2Key Laboratory of Water Cycle and Related Land Surface Processes, Institute of Geographic Sciences and Natural Resources Research, Chinese Academy of Sciences, Beijing, China
3Division of Hydrologic Sciences, Desert Research Institute, Las Vegas, NV 89119, USA
4State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Hohai University, Nanjing, China

The complex dynamics of partial bed-load transport in a series of well-controlled laboratory experiments are explored systematically and simulated by a stochastic model in this study. Flume experiments show that the leading front of bed-load on a 20-meter-long, mixed-size gravel-bed moves anomalously, where the transient transport rate of the accelerating front varies with the observation time scale. In addition, observations show that moving particles may experience bimodal transport (i.e., coexistence of long trapping time and large jump length) related to bed coarsening and the formation of clusters on a heterogeneous gravel-bed, which is distinguished from the traditional theory of hiding-exposing interactions among mixed-size particles. A fractional derivative model is finally applied to characterize the overall behavior of partial bed-load transport, including the coexistence of the sub-diffusion and non-local feature caused by turbulence and the micro-relief within an armor layer.

A03-10 Invited
A finite element method for a class of time-space fractional integro-differential equation
Zhengang Zhao
Department of Fundamental Courses, Shanghai Customs College/ No. 5677, Huaxia West Road, Pudong District, Shanghai City, P.R. China

In this article, a Galerkin finite element approximation for a class of time-space fractional integro-differential equation is considered. In the time direction, a Crank-Nicholson time-stepping is used to approximate the fractional differential term and the product trapezoidal method is employed to treat the temporal fractional integral term. In the spatial direction, the Galerkin finite element method is presented, where the stability analysis, existence and uniqueness analysis of the scheme are discussed. Convergence analysis of semi-discretization scheme and full discretization scheme are also discussed, where the L^2 error bound of finite element accuracy and second order accuracy in time are deduced. Numerical examples are also included to demonstrate the effectiveness of the proposed method.

A03-11 Invited
A numerical method for the fractional Fourier equation
Peng Guo
Department of Mathematics and Physics, Shanghai DianJi University, Shanghai, China

In the last few decades, fractional calculus has got more and more attention, not only in theory but also in applications. In this paper we focus on the fractional Fourier equation and the fractional operator we used is in Caputo sense. An approach based on the fractional convolution method is given. Stability and convergence of the numerical algorithm are proved. A numerical example is presented and compared to the exact solution to verify the theoretical analysis.

A03-12
A general compact ADI scheme for the time-fractional subdiffusion equation in two space dimensions
An Chen1,*, Changpin Li
Department of Mathematics, Shanghai University, Shanghai 200444, China

In this paper, a general compact alternating direction implicit (ADI) scheme is proposed for solving the time-fractional subdiffusion equation in two space dimensions. The established scheme is based on the modified L1 method in time and the compact finite difference method in space. The unique solvability, unconditionally stability and convergence of the scheme are proved. The derived compact ADI scheme is coincident with the one for 2D integer order parabolic equation when the $\beta \to 1$, where $0 < \beta \in (0, 1)$ is the order of the Riemann-Liouville derivative operator. In addition, the general ADI scheme is used to solve the 2D modified fractional diffusion equation, and the corresponding stability and convergence results are also given. Numerical results are provided to verify the theoretical analysis.

A03-13
A novel method to solve two dimensional fractional diffusion equations
Jianshiong Cao1,*, Changpin Li2, YangQuan Chen
1Department of Mathematics, Shanghai University, Shanghai200444, China.
2School of Engineering, UC Merced, Merced, CA 95343, USA

In this talk, we will present an efficient and easy implementation computation platform to numerically solve two dimensional fractional diffusion equations which model anomalous diffusion process. Different from the existed numerical methods, we adopt mobile sensors and actuators in a distributed parameter system to control an anomalous diffusion problem, and develop an efficient solver, called FO-DirFMAS-2D to solve the corresponding fractional diffusion equations. The two dimensional time fractional and time-space fractional diffusion equations are considered, respectively. Finally, several numerical simulation results are shown to testify the efficiency of the new solver.