Mini-symposium A13
Bayesian Inverse Problems and Data Assimilation

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A13-01 Keynote
Structuring Sample-Based Inference for Tomography and Other Linear Inverse Problems
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In the Bayesian setting inverse problems are recast as hierarchical stochastic models, accounting for unknown measurement errors and image properties. Perhaps the most common case is the linear inverse problem with additive Gaussian noise, as often assumed in tomography. Then the use of a low-level Gaussian Markov random field (GMRF) prior model leads to a hierarchical model that may be structured in a number of ways to facilitate sample-based inference.

Most typical is to form the posterior distribution and then perform Markov chain Monte Carlo (MCMC) sampling with random-walk proposal, but this is by no means the most computationally efficient. A sequence of increasingly `blocked' sampling schemes lead to increasing efficiency, depending on which normalizing constants may be computed cheaply. We exploit blocking, and the recent developments in accelerated iterative Gaussian samplers to demonstrate inference for large-scale tomography that outperforms regularization methods in all measures.

A13-02 Invited
Posterior Contraction Rates in Bayesian Inverse Problems
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We discuss Bayesian inverse problems in Hilbert spaces. The focus is on a fast concentration of the posterior probability around the unknown true solution as expressed in the concept of posterior contraction rates. This concentration is dominated by a parameter which controls the variance of the prior distribution. Previous results determine posterior contraction rates based on known solution smoothness. Here we show that an oracle-type parameter choice is possible. This is done by relating the posterior contraction rate to the root mean squared estimation error. In addition we show that the excess probability, which usually is bounded by using Tchebychev's inequality, has exponential decay, at least for a priori parameter choices. These results implement the exponential concentration of Gaussian measures in Hilbert spaces.

A13-03 Invited
Matrix splitting analysis of random-walk Markov chains
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We analyze Metropolis-Hastings algorithms with AR(1) process proposals for sampling from multivariate normal target distributions in high dimensions. This is achieved by making a theoretical connection between AR(1) processes and the matrix splitting formalism for solving systems of linear equations from numerical linear algebra. If the AR(1) process proposal corresponds to a consistent matrix splitting (i.e. the splitting is of the target precision matrix) then acceptance is guaranteed and convergence is governed by the spectral properties of the iteration matrix. However, if the proposal corresponds to an inconsistent matrix splitting (i.e. the splitting is of an effective precision matrix) then convergence is complicated by the Metropolis-Hastings accept/reject step. We are able to show how the expected acceptance probability and squared jump size depends on both the spectral properties of the proposal iteration matrix and the error between the effective target and true target distributions in the case when the number of dimensions tends towards infinity and the splitting matrices are functions of the precision matrix. We also show that discretized Langevin diffusion (e.g. Metropolis-Adjusted Langevin algorithm) and discretized Hamiltonian dynamics (e.g. Hybrid Monte Carlo algorithm) may be written as matrix splittings.

A13-04 Invited
Choice of flaw models in Eddy-Current testing by using Nested Sampling
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In Eddy-Current Testing (ECT), we are interested in obtaining information about the flaws possibly present in the inspected parts. Choice of model is a general problem that one can run into when analyzing the shapes, the number of flaws and furthermore the influence of nuisance parameters, like the lift-off. In the framework of Bayesian inference, we need to calculate the evidences of the concerned models in order to choose between (or among) models. Nested Sampling (NS) shows the possibility of approximating the evidence with reasonable computational cost. This contribution proposes a model choice method based on an improved NS algorithm the aim of which is to get independent samples with hard constraint on the likelihood value in a more efficient way. It works for models who have Gaussian-like likelihood distributions. By analyzing the model evidences approximated by the NS algorithm and the final active samples, this method makes it possible to assign the correct model for the flaw of concern and meanwhile to estimate the corresponding flaw parameters. Simulations have been conducted to validate this method. The results confirm its computational efficiency and model choice reliability insisting the fact that metamodels contribute to the efficiency in the most complex cases.

A13-05 Invited
Multi-Level sequential Monte Carlo samplers
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We consider Bayesian inverse problems associated to elliptic PDEs. In particular, one is interested in expectations of functions w.r.t. the posterior distribution. We consider a sequence of posterior distributions associated to the accuracy of a finite element solver. Typically, the popular multi-level Monte Carlo (MLMC) method (e.g. Giles (2008)) is of use to optimize the variance versus work trade-off. In practice, however, i.i.d. sampling is not possible from the sequence of posteriors. We show how to combine sequential Monte Carlo samplers (Del Moral et al. 2006) with MLMC to obtain efficient estimates of expectations w.r.t. the posterior associated to the most accurate finite element solver. We provide a novel mathematical analysis of the associated estimator, recovering, under assumptions, the “optimal” regime.

A13-06 Invited
Data-driven tight frame and sparse image modelling
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Sparse image modeling has been one basic tool in solving ill-posed inverse problems arising from imaging sciences. Sparse image model assumes that image of interest has a good sparse approximation under a certain system. One widely used such systems in image recovery are wavelet tight frames. There have been enduring efforts on seeking wavelet tight frames under which a certain class of functions or images can have a good sparse approximation. However, the structure of images varies greatly in practice and a system working well for one type of images may not work for another. In this paper, we presents a numerical method that derives a discrete tight frame system from the input image itself to provide a better sparse approximation to the input image. Such an adaptive tight frame construction scheme is applied to image de-noising by constructing a tight frame tailored to the given noisy data.

A13-07 Invited
Multilevel Markov Chain Monte Carlo method for Bayesian inverse problems
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The Bayesian approach to inverse problems, in which the posterior probability distribution on an unknown field is sampled for the purposes of computing posterior expectations of quantities of interest, is starting to become computationally feasible for partial differential equation (PDE) inverse problems. Balancing the sources of error arising from finite-dimensional approximation of the unknown field, the PDE forward solution map and the sampling of the probability space under the posterior distribution are essential for the design of efficient computational Bayesian methods for PDE inverse problems. We study Bayesian inversion for a model elliptic PDE with an unknown diffusion coefficient. We provide complexity analysis of Markov chain Monte Carlo (MCMC) methods for numerical evaluation of expectations under the Bayesian posterior distribution, given data δ, in particular bounds on the overall work required to achieve a prescribed error level ε. We first bound the computational complexity of ‘plain’ MCMC, based on combining MCMC sampling with linear complexity multi-level solvers for elliptic PDE. The work versus accuracy bounds show that the complexity of this approach can be quite prohibitive. We then present a novel multi-level Markov chain Monte Carlo strategy which utilizes sampling from a multi-level discretization of the posterior and the forward PDE. The strategy achieves an optimal complexity that is essentially equal to that for performing only one step of the plain MCMC.

This is a joint work with Ch, Schwab (ETH, Zurich) and A. M. Stuart (Warwick)