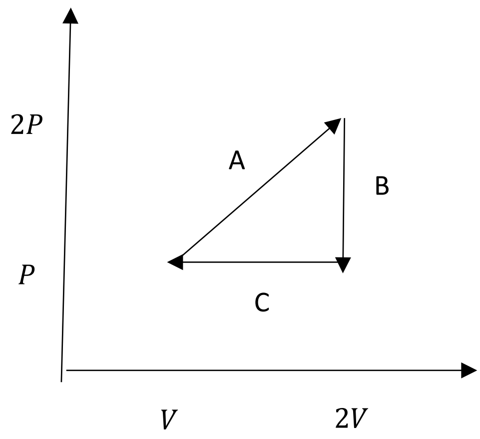
PC2135 Thermodynamics and Statistical Mechanics

Midterm test Friday 8 Mar (close book), 1 hour 45 minutes

1. Calculate the average volume occupied by one molecule for an ideal gas at room temperature (300 K) and atmospheric pressure (1.013×10^5 N/m^2). Then take the cube root to get an estimate of the average distance between molecules. How does this distance compare to the size of a small molecule like N_2 or H_2O?

2. A diatomic ideal gas near room temperature is made to undergo the cyclic process shown in the figure. They form a triangular closed loop by paths A, B, and C. Compute for each of the steps A, B, and C, (a) work done on the gas; (b) the change in the energy content of the gas; (c) the heat added to the gas. Express the answers in P and V only.

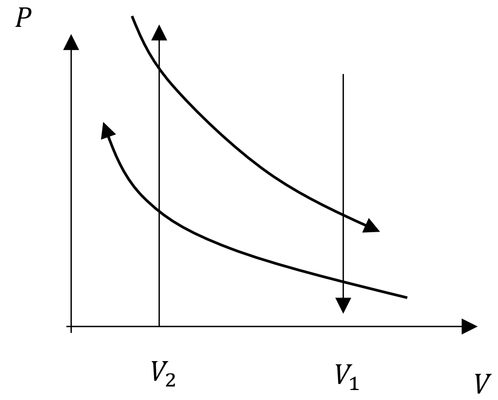


3. Consider a two-state paramagnet where each spin can only be up or down. Let the total number of spins be N and the number of the up spins be $N_\uparrow$. (a) Give the multiplicity of the paramagnet in terms of N and $N_\uparrow$. (b) Give the expression for entropy assuming N and $N_\uparrow$ are large (i.e. use the Sterling's approximation). (c) Rewrite the entropy as a function of temperature T. To do this, you need to relate the energy to the number of up spins by $U = \mu B(N - 2N_\uparrow)$ where μ is magnetic moment per spin, and B is the magnetic induction.

4. Experimental measurements of the heat capacity of aluminum at low temperatures (blow 50 K) can be fit to the formula $C_V = aT + bT^3$, where C_V is the heat capacity of one mole of aluminum, and constants a and b are approximately $a = 0.00135$ J/K^2 and $b = 2.48 \times 10^{-5}$J/K^4.

a. From this data, find a formula for the entropy of a mole of aluminum as a function of temperature.
b. Evaluate your formula at $T = 1$ K and $T = 10$ K, expressing your answers both in conventional units (J/K) and as unitless numbers (i.e. in units of the Boltzmann constant, $k = 1.381 \times 10^{-23}$ J/K).

5. The Otto cycle consists four steps as shown on the P-V diagram, two vertical lines (isocharic) and two adiabatic lines, forming a loop. Let the volumes of the two vertical lines be V_1 and V_2 with $V_2 < V_1$. Show that the ideal Otto cycle using ideal gas a substance has the efficiency $e = \frac{W}{Q_h} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$, where the exponent γ is in $PV^{\gamma} =$ const for adiabatic process.



Formula sheet

Ideal gas law: $PV = NkT$

Equipartition theorem: $U = \frac{f}{2}NkT$

First law: $\Delta U = Q + W$

Second law: $\Delta S \geq Q/T$

Thermodynamic identity: $dU = TdS - PdV + \mu dN$

Physical constants: $k = 1.381 \times 10^{-23}$J/K, $N_A = 6.022 \times 10^{23}$, $e = 1.602 \times 10^{-19}$ C

Adiabatic process: $PV^{\gamma} =$ const.

Boltzmann principle: $S = k \ln \Omega$

Multiplicity of Einstein solid: $\Omega = \frac{(q+N-1)!}{q!(N-1)!}$

Sterling formula: $\ln N! \approx N \ln N - N$

Sackur-Tetrode formula: $S = Nk\left[\ln\left(\frac{V}{N^{5/2}}\left(\frac{4\pi mU}{3h^2}\right)^{3/2}\right) + \frac{5}{2}\right]$

Carnot efficiency: $e = \frac{W}{Q_h} = 1 - T_c/T_h$