NATIONAL UNIVERSITY OF SINGAPORE

PC2135 Thermodynamics and Statistical Mechanics

(Semester II: AY 2022-23)

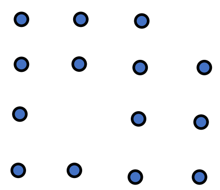
Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Please write only your student number. Do not write your name.
2. This assessment paper contains five questions. It comprises three printed pages.
3. Students are required to answer ALL the questions. Questions carry equal marks.
4. The answers are to be written on the answer books.
5. Students should write the answers for each question on a new page.
6. This is a CLOSED BOOK examination.
7. Non-programmable calculators are allowed.

1. Thermal photons confined in a volume of V of blackbody cavity has energy U. The photon gas generates a pressure according to $P = \frac{U}{3V}$.
   a. Show that $\frac{TS}{U} = \frac{4}{3}$, based on the Euler equation.
   b. Show that an adiabatic process for photon gas is given by $UV^{1/3}$= const.
   c. Since in a reversible adiabatic process entropy is fixed, the result of part b means that the entropy must be a function of $x = UV^{1/3}$. Determine the functional form $S(x)$ of a photon gas, by thermodynamic means.

2. Schottky defects are missing atoms in a crystal lattice. See the illustration below for a case of $N = 16$ sites with $n = 2$ missing atoms. We assume each missing atom costs an energy ϵ.
   a. For a lattice with a total N sites and n missing atoms or vacancies, what is the multiplicity Ω? That is, how many ways can we arrange the defect sites?
   b. Compute the entropy of the system, and determine the ratio n/N as a function of ϵ and temperature T. Here, we can use Sterling's approximation $\ln N! = N \ln N - N$.



3. In a distance star, it is found that in the atmosphere the amount of hydrogen atoms in the first excited states (2s, $2p_x$, $2p_y$, $2p_z$) comparing to the ground state is 1.6 $\times 10^{-5}$ to 1. For hydrogen atom, the ground state has energy -13.6 eV, and the first excited states -3.4 eV with a four-fold degeneracy.
   a. Estimate the temperature of the star.
   b. Calculate the entropy S of one hydrogen atom in the star as a two-level system (disregarding other degrees of freedoms).

4. A non-ideal gas system has been calculated to have the following Helmholtz free energy, $F = -\frac{aN^2}{V} - NkT \ln(V - bN) - \frac{3}{2} NkT \ln\frac{T}{c} + \text{const}$. Here a, b, c are some constants.
   a. Based on the free energy given, determine the equation of state, i.e., a relation relating pressure P, volume V, and temperature T.
   b. The equation of state in part a is not quite correct below a critical temperature, and Maxwell's construction is needed. Determine this critical temperature T_c in terms of the model constants.

c. Explain what is a Maxwell's construction, and what is the fundamental principle used for the construction.

5. Consider a one-dimensional particle-in-a-box quantum problem. The energy levels of a single particle are given as $E_n = h^2 n^2/(8mL^2)$ where h is the Planck constant, m is mass of the particle, L is the length of the box, the quantum number $n = 1, 2, 3, \ldots$
   a. Consider the lowest possible temperature $T = 0$, so that the system of N particles as a whole is in its ground state. Calculate the chemical potential $\mu(T = 0) = \epsilon_F$ if the particles are fermionic electrons with spin degeneracy (spin up and spin down has the same energy).
   b. Repeat the same calculation for chemical potential for spinless bosonic particles.
   c. For the one-dimension electron problem of part a, at what temperature Boltzmann distribution becomes valid? Express the temperature in terms of the model parameters, as well as give a numerical value in kelvin assuming the 1D electrons are spaced at a distance 10^{-10}m apart.

--The END -- [WJS]

Physical Constants

$$k = 1.381 \times 10^{-23}\ \mathrm{J/K}$$
$$= 8.617 \times 10^{-5}\ \mathrm{eV/K}$$
$$N_A = 6.022 \times 10^{23}$$
$$R = 8.315\ \mathrm{J/mol{\cdot}K}$$
$$h = 6.626 \times 10^{-34}\ \mathrm{J{\cdot}s}$$
$$= 4.136 \times 10^{-15}\ \mathrm{eV{\cdot}s}$$
$$c = 2.998 \times 10^{8}\ \mathrm{m/s}$$
$$G = 6.673 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$$
$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$
$$m_e = 9.109 \times 10^{-31}\ \mathrm{kg}$$
$$m_p = 1.673 \times 10^{-27}\ \mathrm{kg}$$

Unit Conversions

$$1\ \mathrm{atm} = 1.013\ \mathrm{bar} = 1.013 \times 10^{5}\ \mathrm{N/m^2}$$
$$= 14.7\ \mathrm{lb/in^2} = 760\ \mathrm{mm\,Hg}$$
$$(T \text{ in } {}^\circ\mathrm{C}) = (T \text{ in K}) - 273.15$$
$$(T \text{ in } {}^\circ\mathrm{F}) = \tfrac{9}{5}(T \text{ in } {}^\circ\mathrm{C}) + 32$$
$$1\ {}^\circ\mathrm{R} = \tfrac{5}{9}\ \mathrm{K}$$
$$1\ \mathrm{cal} = 4.186\ \mathrm{J}$$
$$1\ \mathrm{Btu} = 1054\ \mathrm{J}$$
$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$$
$$1\ \mathrm{u} = 1.661 \times 10^{-27}\ \mathrm{kg}$$