PC1221 Fundamentals of Physics I

Semester-1, AY08/09

Lectures 1 and 2

Introduction, Historical Overview and Measurement

A/Prof Tay Seng Chuan

Ground Rules

- Switch off your handphone and pager
- Switch off your laptop computer and keep it
- No talking while lecture is going on
- No gossiping while the lecture is going on
- Raise your hand if you have question to ask
- Be on time for lecture
- Be on time to come back from the recess break to continue the lecture
- Bring your lecturenotes to lecture

Introduction of Lecturer

A/Prof Tay Seng Chuan
Tel: 65168752
Email: scitaysc@nus.edu.sg
Office: S16-02, Dean's Office at Level 2
URL: http://www.physics.nus.edu.sg/~phytaysc

Current Duties in NUS:

Day: Associate Professor (Physics), Head of Faculty IT Unit (Science), Coordinator for SM2/SM3 Programmes (MOE), Associate Director (Office of Alumni Relations) representing Temasek Hall. I am also a regular project mentor of MOE Gifted Education Programme.

Night: Assistant Master of Temasek Hall, Resident Fellow (Block E).

Consultation for PC1221

- Day: S16-02, Dean's Office. By email (scitaysc@nus.edu.sg) or telephone call (65168752) for appointment. It is better to give a telephone call before you come to see me. I would appreciate that you will indentify yourself when you call me.
- Night: Temasek Hall, Block E, Unit E100. Please make appointment in advance if you need to consult me at night so that I will be waiting for you.

Assessments for this module

- 5 sets of lab report – 20%
- 2 Tests (Closed Book) – 20%
- Final Exam (Closed Book) – 60%

Textbooks

Main text: Physics for Scientists and Engineers (Volume 1), – Serway Jewett, ISBN 0-534-40956-3, Thomson Brooks/Cole. Available at NUS Coop, behind LT27.

Complementary books:

(i) Physics – Giancoli, ISBN 0-13-611971-9, Prentice Hall.

(ii) College Physics – Giambattista Richardson Richardson, ISBN-13 878-0-07-110608-5, McGraw Hall.

(iii) physics for scientists and engineers – Paul A. Tippler, ISBN 1-5259- 673-2, Freeman Worth.(18 copies at **$30/each** (usual price: $48) available from Dr Roland Su (Tel: 65164930, email: scisur@nus.edu.sg; Office: S16-09.)

Contents

- This module is for students **without** Physics qualifications at GCE “A” level or its equivalent. You should not take this module if you have “A” level Physics or its equivalent
- In this module I will introduce you to only two main topics in Physics:
  - Mechanics
  - Thermodynamics

How we are going to do it

- 48 Lecture hours (Dr Roland Su will cover me in my absence.)
- 5 Laboratory sessions (3 hours/session)
- 5 Tutorial sessions (1 hour/session)
- 2 Tests
- Final exam

Timetable

Semester 1	Mon 4 Aug - Sat 6 Dec 2008	18 weeks
Orientation Week:	Mon 4 Aug - Sat 9 Aug 2008 (a)	1 week
Instructional Period: Week 1	Mon 11 Aug – Fri 15 Aug 2008	6 weeks
Week 2	Mon 18 Aug – Fri 22 Aug 2008	
Week 3	Mon 25 Aug – Fri 29 Aug 2008	
Week 4	Mon 1 Sep – Fri 5 Sep 2008	
Week 5	Mon 8 Sep – Fri 12 Sep 2008	
Week 6	Mon 15 Sep – Fri 19 Sep 2008	
Recess Week:	Sat 20 Sep – Sun 28 Sep 2008	1 week
Instructional Period: Week 7	Mon 29 Sep – Fri 3 Oct 2008 (b)	7 weeks
Week 8	Mon 6 Oct – Fri 10 Oct 2008	
Week 9	Mon 13 Oct – Fri 17 Oct 2008	
Week 10	Mon 20 Oct – Fri 24 Oct 2008	
Week 11	Mon 27 Oct – Fri 31 Oct 2008 (c)	
Week 12	Mon 3 Nov – Fri 7 Nov 2008	
Week 13	Mon 10 Nov – Fri 14 Nov 2008	
Reading Week:	Sat 15 Nov - Fri 21 Nov 2008	1 week
Examination:	Sat 22 Nov - Sat 6 Dec 2008	2 weeks
Vacation:	**Sun 7 Dec 2008 - Sun 11 Jan 2009** (d), (e) & (f)	**5 weeks**

Schedule

- Lecture on Tuesday and Friday, 10am – 12noon, LT27.
- Tutorial and lab for SM2 scholars have been pre-fixed.
- Tutorial and lab for Engineering students have been pre-fixed.
- Tutorial and labs for other NUS students (Science, SoC, FASS, etc) will be arranged based on enrollment figure (Tuesday and Friday afternoon).
- Data analysis (computer class) will be conducted on week-2 at S13-04. All students are to attend and will have to hand in the assignments on time. For the subsequent weeks, please go to Physics year-1 lab to do Physics experiments and this is to be alternated with Physics tutorial.

Resources

- IVLE (Integrated Virtual Learning Environment): http://ivle.nus.edu.sg

  You can post questions there for discussion. All students are encouraged to contribute their answers.

  (IVLE is solely for academic discussion purpose. I expect a certain degree of courtesy in this discussion group. Personal agenda is not allowed. I always say "freedom is not free play.")
- Online resource: www.aw.com/young11

Expectations

- Attend all lectures, labs and tutorials
- Participate in IVLE discussion at least twice a week
- Lectures
  - Spend 30 mins reading the textbook and lecture notes on the topics to be covered in the upcoming lecture.
  - Spend 60 mins reading and practicing the questions in the textbook after the lecture.
- Tutorials
  - Prepared fully written answers for most of the questions. You can be asked to show your solutions so please do your homework.

Expectations

- Laboratory
  - Read the Lab scripts before your scheduled sessions
  - Come with a spirit of wanting to learn
  - **Plagiarism is a serious offence**
- Course website
  - Regularly access course website for up-to-date materials

Arrangement for **SM2** Students (Please refer to the printouts given last week.)

PC1221 PRACTICALS & TUTORIALS SCHEDULE

Data Analysis session will be conducted on 21 August 2008 9am-12pm at CSD laboratories. Attendance is compulsory. Seating arrangement is the same as C-Programming labs.

Experiment 1: Simple measurements
Experiment 2: Inclined plane
Experiment 3: Newton's second law
Experiment 4: Force of gravity
Experiment 5: Thermal expansion

Date / Group	28 Aug	4 Sept	11 Sept	18 Sept	2 Oct	9 Oct	16 Oct	23 Oct	30 Oct	6 Nov
A1	Expt. 1	Tut. 1	Expt. 2	Tut. 2	Expt. 3	Tut. 3	Expt. 4	Tut. 4	Expt. 5	Tut. 5
A2	Expt. 2		Expt. 3		Expt. 4		Expt. 5		Expt. 1	
A3	Expt. 3		Expt. 4		Expt. 5		Expt. 1		Expt. 2	
A4	Expt. 4	Tut. 1	Expt. 5	Tut. 2	Expt. 1	Tut. 3	Expt. 2	Tut. 4	Expt. 3	Tut. 5
A5	Expt. 5		Expt. 1		Expt. 2		Expt. 3		Expt. 4	
B1	Tut. 1	Expt. 1	Tut. 2	Expt. 2	Tut. 3	Expt. 3	Tut. 4	Expt. 4	Tut. 5	Expt. 5
B2		Expt. 2		Expt. 3		Expt. 4		Expt. 5		Expt. 1
B3		Expt. 3		Expt. 4		Expt. 5		Expt. 1		Expt. 2
B4	Tut. 1	Expt. 4	Tut. 2	Expt. 5	Tut. 3	Expt. 1	Tut. 4	Expt. 2	Tut. 5	Expt. 3
B5		Expt. 5		Expt. 1		Expt. 2		Expt. 3		Expt. 4

Arrangement for **Engineering** Students (please refer to IVLE for groupings) taking only PC1221 without taking PC1222

Tutorial (even week)

Time / Day	9 - 10	10 - 11	11 - 12	12 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Monday						ST1, ST5		
Tuesday						ST2, ST6		
Wednesday								
Thursday						ST3, ST7		
Friday						ST4, ST8		

ST1, 5 - Venue: Block S11-03-01 Physics Lab Instruction Room
ST2, 3, 4, 6,7,8 - Venue: Block S13-04-14 CSD Lab 2

Lab (odd weeks)

Time / Day	9 - 10	10 - 11	11 - 12	12 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Monday							SA1, SA5	
Tuesday							SA2, SA6	
Wednesday								
Thursday							SA3, SA7	
Friday							SA4, SA8	

Venue: Block S12-04-02 Level 1 Physics Lab

Arrangement for **Engineering** Students (please refer to IVLE for groupings) taking both PC1221 and PC1222

Tutorial (odd week)

Time / Day	9 - 10	10 - 11	11 - 12	12 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Monday						ST1, ST5		
Tuesday						ST2, ST6		
Wednesday								
Thursday						ST3, ST7		
Friday						ST4, ST8		

ST1, 5 - Venue: Block S11-03-01 Physics Lab Instruction Room
ST2, 3, 4, 6,7,8 - Venue: Block S13-04-14 CSD Lab 2

Lab (even weeks)

Time / Day	9 - 10	10 - 11	11 - 12	12 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Monday							SA1, SA5	
Tuesday							SA2, SA6	
Wednesday								
Thursday							SA3, SA7	
Friday							SA4, SA8	

Venue: Block S12-04-02 Level 1 Physics Lab

Arrangement for Non-Engineering NUS Students

Please sign up your tutorial class and lab class at Physics first year lab (S12-04) after this lecture.

Tutorial (odd weeks):
Tuesday or Friday: 2 to 3pm.

Day \ Time	9 - 10	10 - 11	11 - 12	12 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Monday								
Tuesday						ST2, ST6		
Wednesday								
Thursday								
Friday						ST4, ST8		

Lab (even weeks):
Tuesday or Friday: 2 to 5pm.

Day \ Time	9 - 10	10 - 11	11 - 12	12 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Monday								
Tuesday						SA2, SA6		
Wednesday								
Thursday								
Friday						SA4, SA8		

1	ANG CHIN HUAT JASON
2	AVERILLA PAUL STEPHEN REMPOLA
3	CHUNG YI FANG MAVIS
4	ERIK JENS CARL HENRIK AABERG
5	HAN WUGUANG
6	HO PEH SUAN
7	HOANG THAI DUONG
8	JOANNA TAN HUI JUAN
9	KANG HWEE
10	LEE CHEE YUAN
11	LEE HONG WEI
12	LEI XIXI
13	LI LIQIN
14	LIN YUNYAO
15	LIU YUE
16	LOH CHIN KANG BERNARD
17	MARINAH MUKHTAR TOH ABDULLAH
18	MATHEW NG MUN HOOU
19	OU YANG XIULING
20	SARAH WHITE LARSEN
21	TAN JING PING
22	YAP KENG CHUAN
23	YEO KUN SONG

Week 2: Data Analysis Lab will be conducted in CSD Computer Lab (S13-04-16) (refer to IVLE)

Engineering and non-Engineering NUS Students:
(1) Tuesday, 19 August, 2-5pm
(2) Wednesday, 20 August, 2-5pm
(3) Thursday, 21 August, 2-5pm
(4) Friday, 22 August, 2-5pm
Go according to your Lab day. Monday's group can go to any day for Data Analysis Computer Lab.

SM2 Students: Thursday, 21 August, 9am to 12 noon

Physics Tutorial and Physics Lab will start on week-3.

Support Team at Physics Year-1 Lab

Mdm Pang Teng Jar
Lab In Charge

Mdm Cheong, Elaine

Mr Foong Chee Kong

Mdm Koh Siew Lee

Teaching Team

Mr. Andreas Dewanto
Office: S13-02-07
Tel: (65)-6516-2817
Fax: (65)-6777-6126
Email: phyda@nus.edu.sg
Tutor for SM2 and Engineering Students

A/Prof Tay Seng Chuan
Lecturer for all, and Tutor for Science and SoC Students

Mr Kenneth Hong
Office: S13-04-03
Tel: (65)-6516-2631
Fax: (65)-6777-6126
Email: phyhcmk@nus.edu.sg
Lab Instructor (Assisted by Honours Year and Graduate Students)

Dr Roland Su will cover me in my absence.

End of Introduction

Any question?

Historical Overview

- Physics is a fundamental science
  - concerned with the basic principles of the Universe
  - foundation of other physical sciences
- Physics is divided into six major areas
  - Classical Mechanics
  - Relativity
  - Thermodynamics
  - Electromagnetism
  - Optics
  - Quantum Mechanics

Classical Physics

- Mechanics and electromagnetism are basic to all other branches of classical physics
- Classical physics developed before 1900
  - Our study will start with Classical Mechanics
    - Also called Newtonian Mechanics

Classical Physics, cont

Includes

- Mechanics
  - Major developments by Newton (1642 - 1727), and continuing through the latter part of the 19th century
- Thermodynamics
- Optics
- Electromagnetism
  - All of these were not developed until the latter part of the 19th century mainly because the apparatus was too crude or unavailable.

Modern Physics

- Began near the end of the 19th century
- Phenomena that could not be explained by classical physics, eg, when the speed of an particle is extremely fast (near speed of light)
- Includes theories of relativity (Albert Einstein, 1879-1955) and quantum mechanics

Classical Mechanics Today

- Still important in many disciplines, eg, bio-physics
- There is a wide range of phenomena that can be explained with classical mechanics, eg, traffic congestion along highways
- Many basic principles carry over into other phenomena and applications, eg, high speed computation, unmanned planetary explorations, manned moon landings, etc.
- Conservation Laws also apply directly to other areas such as in global warming, tidal wave etc.

Objective of Physics

- To find the limited number of fundamental laws that govern natural phenomena
- To use these laws to develop theories that can predict the results of future experiments
- Express the laws in the language of mathematics
- We want to put down the ideas in concrete form (in written words).

Theory and Experiments

- They should complement each other
- When a discrepancy occurs, theory may be modified, and usually not the other way around. We should not cheat in our experiment. It is also now a criminal offence to cheat in research work (misuse of public fund).

http://www.telegraph.co.uk/news/main.jhtml?xml=/news/2005/12/24/woosuk24.xml

Embryo cloning cheat resigns in disgrace

By Roger Highfield Science Editor
Last Updated: 1:07am GMT 24/12/2005

The world's most successful cloning scientist, Prof Hwang Woo-suk, who was hailed as a superstar with "God's hand" in his native South Korea, has resigned in disgrace.The furore that erupted yesterday over how his team partly faked results will send shock waves around the scientific world, damage the image of biotechnology and cast a shadow over rival British efforts to develop the next generation of medicine.

Prof Hwang Woo-suk apologises to South Korea

Hwang made headlines last year when he unveiled the first cloned human embryo. This year he published an apparent tour de force, reporting the creation of more than 30 cloned human embryos and 11 lines of stem cells which could be grown into any of the 200 cell types found in the body.

The work marked the start of what many scientists believed would be a revolution, with stem cells being grown from patients to treat a vast range of diseases.

Pro-life groups condemned the research as "repugnant manipulation" and "trivialisation" of a human life.

Never cheat in our experiment!! No plagiarism !!

Theory and Experiments, cont

- Theory may apply to limited conditions only. It may not be able to explain all physical phenomena.
  - Example: Newtonian Mechanics is confined to objects traveling slowly with respect to the speed of light
- We should try to develop a more general theory. Why?

A specific explanation to a particular case may not be applicable to other cases.

Quantities Used

- In mechanics, three *basic quantities* are used
  - Length
  - Mass
  - Time
- We will also use *derived quantities*
  - Derived quantities can be expressed in terms of basic quantities

Standards of Quantities

- Standardized systems
  - agreed upon by some authority, usually a governmental body
- SI – Systéme International
  - agreed to in 1960 by an international committee
  - SI system is used in this module

Length

- Units
  - SI – meter, m
- Defined in terms of a meter – the distance traveled by light in a vacuum during a given time of 1/299792458 second (announced in October 1983).

Table 1.1

Approximate Values of Some Measured Lengths

	Length (m)
Distance from the Earth to the most remote known quasar	1.4×10^{26}
Distance from the Earth to the most remote normal galaxies	9×10^{25}
Distance from the Earth to the nearest large galaxy (M 31, the Andromeda galaxy)	2×10^{22}
Distance from the Sun to the nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of the Earth about the Sun	1.50×10^{11}
Mean distance from the Earth to the Moon	3.84×10^{8}
Distance from the equator to the North Pole	1.00×10^{7}
Mean radius of the Earth	6.37×10^{6}
Typical altitude (above the surface) of a satellite orbiting the Earth	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	$\sim 10^{-4}$
Size of cells of most living organisms	$\sim 10^{-5}$
Diameter of a hydrogen atom	$\sim 10^{-10}$
Diameter of an atomic nucleus	$\sim 10^{-14}$
Diameter of a proton	$\sim 10^{-15}$

91 m

Mass

- Units
  - SI – kilogram, kg
- Defined in terms of a kilogram, based on a specific cylinder made of platinum-iridium alloy kept at the International Bureau of Standards in France

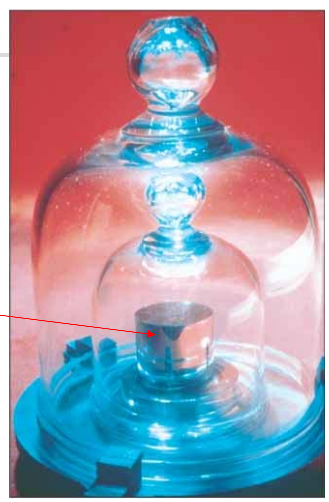


Table 1.2

Masses of Various Objects (Approximate Values)

	Mass (kg)
Observable Universe	$\sim 10^{52}$
Milky Way galaxy	$\sim 10^{42}$
Sun	1.99×10^{30}
Earth	5.98×10^{24}
Moon	7.36×10^{22}
Shark	$\sim 10^{3}$
Human	$\sim 10^{2}$
Frog	$\sim 10^{-1}$
Mosquito	$\sim 10^{-5}$
Bacterium	$\sim 1 \times 10^{-15}$
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

~100 kg

Time

- Units
  - seconds, s
- Defined in terms of the oscillation of radiation from a cesium atom (stable for 20 million years)

(Cesium is used to make atomic clock)

Table 1.3

Approximate Values of Some Time Intervals

	Time Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day (time interval for one revolution of the Earth about its axis)	8.6×10^{4}
One class period	3.0×10^{3}
Time interval between normal heartbeats	8×10^{-1}
Period of audible sound waves	$\sim 10^{-3}$
Period of typical radio waves	$\sim 10^{-6}$
Period of vibration of an atom in a solid	$\sim 10^{-13}$
Period of visible light waves	$\sim 10^{-15}$
Duration of a nuclear collision	$\sim 10^{-22}$
Time interval for light to cross a proton	$\sim 10^{-24}$

$6.3\text{x}10^8/(3600\text{x}24\text{x}365)=19.98$ years

Calculate the ages of your parents in seconds.

Summary d1-1

Number Notation

- When writing out numbers with many digits, it is recommended that you space them in groups of three
  - No commas (comma is used in financial figure, eg: $45,000)
- Examples:
  - 25 100
  - 5.123 456 789 12

Reasonableness of Results

- When solving a problem, we need to check our answer to see if it seems reasonable. Eg: Your weight of 62.5345456 kg is an unnecessarily overloaded figure. Simply 62.5 kg will do!!

Systems of Measurements, cont

- US Customary
  - Length is measured in feet
  - Time is measured in seconds
  - Mass is measured in slugs

Prefixes

- The prefixes can be used with any base units
- They are multipliers of the base unit
- Examples:
  - 1 mm = 10^{-3} m
  - 1 mg = 10^{-3} g

Table 1.4

Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-24}	yocto	y
10^{-21}	zepto	z
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{21}	zetta	Z
10^{24}	yotta	Y

Model Building

- A ***model*** is a system of physical components
  - Identify the components
  - Make predictions about the behavior of the system
    - The predictions will be based on interactions among the components and/or
    - Based on the interactions between the components and the environment
    - Model is evolving

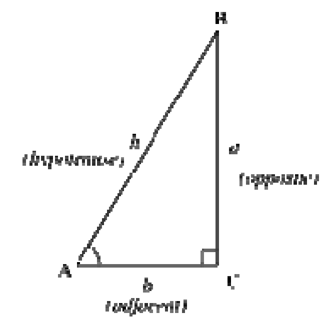
Model of Trigonometric Functions

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{h}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{h}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

$$\tan A = \frac{\sin A}{\cos A}$$



Question: A high fountain of water is located at the center of a circular pool as in Figure. Not wishing to get his feet wet, a student walks around the pool and measures its circumference to be 15.0 m. Next, the student stands at the edge of the pool and uses a protractor to gauge the angle of elevation of the top of the fountain to be 55.0°. How high is the fountain?

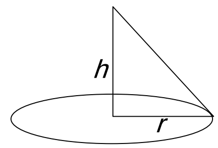
Answer:

$$2\pi r = 15.0\ \mathrm{m}$$

$$r = 2.39\ \mathrm{m}$$

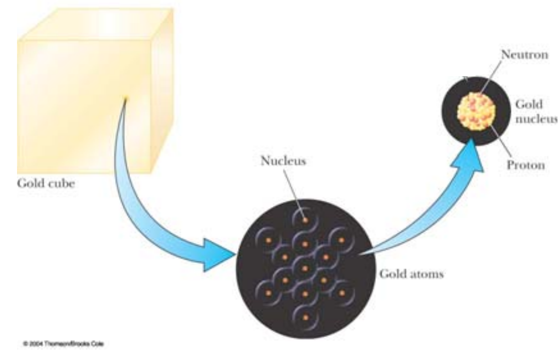
$$\frac{h}{r} = \tan 55.0°$$

$$h = (2.39\ \mathrm{m})\tan(55.0°) = \boxed{3.41\ \mathrm{m}}$$



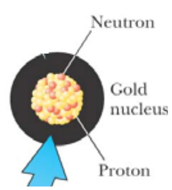
Models of Matter

- Some Greeks thought matter is made of atoms
- JJ Thomson (1897) found electrons and showed atoms had structure
- Rutherford (1911) discovered central nucleus surrounded by electrons



Models of Matter, cont

- Nucleus has structure, containing protons and neutrons
  - Number of protons gives atomic number
  - Total number of protons and neutrons gives mass number
- Protons and neutrons are made up of quarks



- There are 6 types of quarks, named as up, down, strange, charmed, bottom and top.
- The up, charmed and top quarks have electric charges of +2/3 that of the proton.
- The down, strange, and bottom quarks have charges of -1/3 that of the proton.
- Each proton has 2 up quarks and 1 down quark, and each neutron has 2 down quark and 1 up quark.

Can you explain why each proton has +1 unit of charge, and neutron has no charge?

Modeling Technique

- Important technique is to build a model for a problem
  - Identify a system of physical components for the problem
  - Make predictions of the behavior of the system based on the interactions among the components and/or the components and the environment

Density

- Density is an example of a ***derived*** quantity
- It is defined as mass per unit volume

$$\rho \equiv \frac{m}{V}$$

- Units are kg/m^3

Table 1.5

Densities of Various Substances

Substance	Density ρ (10^3 kg/m^3)
Platinum	21.45
Gold	19.3
Uranium	18.7
Lead	11.3
Copper	8.92
Iron	7.86
Aluminum	2.70
Magnesium	1.75
Water	1.00
Air at atmospheric pressure	0.0012

Weight of about 15 soldiers

Atomic Mass

- The atomic mass is the total number of protons and neutrons in the element
- Can be measured in ***atomic mass units***, u
  - 1 u = 1.6605387 x 10^{-27} kg

Basic Quantities and Their Dimension

- Dimension has a specific meaning - it denotes the physical nature of a quantity
- Dimensions are denoted with square brackets
  - Length [L]
  - Mass [M]
  - Time [T]

Dimensional Analysis

- Technique to check the correctness of an equation or to assist in deriving an equation
- Dimensions (length, mass, time, combinations) can be treated as algebraic quantities
  - add, subtract, multiply, divide
- Both sides of equation must have the same dimensions

Dimensional Analysis, cont.

- Cannot give numerical factors: this is its limitation
- Dimensions of some common quantities are given below

Table 1.6

Units of Area, Volume, Velocity, Speed, and Acceleration

System	Area (L^2)	Volume (L^3)	Speed (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
U.S. customary	ft^2	ft^3	ft/s	ft/s^2

Symbols

- The symbol used in an equation is not necessarily the symbol used for its dimension

  Eg: $x = v \times t$, i.e., distance = velocity × time
- Some quantities have one symbol used consistently
  - For example, time is t
  - Some quantities have many symbols used, depending upon the specific situation. For example, lengths may be x, y, z, r, d, h, etc.

Dimensional Analysis, example

- Given the equation: $x = \frac{1}{2} at^2$, where a is acceleration (v / t, i.e., (L/T)/T), t is time, and x is distance
- Check dimensions on each side:

$$L = \frac{L}{T^2} \cdot T^2 = L$$

- The T^2's cancel each other, leaving L for the dimensions of each side
  - The equation is dimensionally correct
  - There are no dimensions for the constant ½.

Example. Newton's law of universal gravitation is represented by

$$F = \frac{GMm}{r^2}$$

Here F is the gravitational force exerted by one small object on another, M and m are the masses of the objects, and r is a distance. Force has the SI units kg·m/s^2. What are the SI units of the proportionality constant G?

Answer: Inserting the proper units for everything except G,

$$\frac{[\mathrm{m}]^2}{[\mathrm{kg}]^2} \times \left[\frac{\mathrm{kg\ m}}{\mathrm{s}^2}\right] = \frac{G[\mathrm{kg}]^2}{[\mathrm{m}]^2} \times \frac{[\mathrm{m}]^2}{[\mathrm{kg}]^2}$$

Question. If an equation is dimensionally correct, does it always mean that the equation is correct?

Answer:

If an equation is dimensionally correct, it does not always mean that the equation is correct. E.g.,
1 "monkeys" = 4 "monkeys" is dimensionally correct, but 1 is not equal to 4.

For an equation to be correct, it must first be dimensionally correct. E.g., 6 "monkeys" = 6 "apples" is dimensionally not correct so the equation is not correct.

Conversion of Units

- When units are not consistent, you may need to convert them to appropriate ones
- Units can be treated like algebraic quantities that can cancel each other
- See the inside of the back cover for an extensive list of conversion factors, eg,

  1 in. = 2.54 cm, 1 slug = 14.59 kg,

  1 gal = 3.786 L, etc.

Conversion

- Always include units for every quantity, you can carry the units through the entire calculation
- Multiply original value by a ratio equal to one
- Example $15.0\,\text{in} = ?\,\text{cm}$

$$15.0\,\text{in}\left(\frac{2.54\,\text{cm}}{1\,\text{in}}\right) = 38.1\,\text{cm}$$

Conversion

- Cross Multiplying Reasoning
  Example

1 in = 2.54 cm
15 in = x cm

$1 \times x = 2.54 \times 15$

$x = 38.1$ cm

Uncertainty in Measurements

- There is uncertainty in every measurement -- this uncertainty carries over through the calculations
  - need a technique to account for this uncertainty
- We will use rules for significant figures to approximate the uncertainty in results of calculations

Significant Figures

- A significant figure is one that is reliably known
- Zeros may or may not be significant
  - Eg, How accurate is 1500 kg in a measurement? Is the accuracy up to 1 kg, 10 kg, or 100 kg?
  - Those zeros used to position the decimal point are not significant. Eg, 0.00075
  - To remove ambiguity, use scientific notation
- In a measurement, the significant figures include the first estimated digit

Significant Figures, examples

- 0.0075 m has 2 significant figures
  - The leading zeros are placeholders only
  - Can write in scientific notation to show more clearly: 7.5×10^{-3} m for 2 significant figures
- 10.0 m has 3 significant figures
  - The decimal point gives information about the reliability of the measurement
- 1500 m is ambiguous
  - Use 1.5×10^3 m for 2 significant figures
  - Use 1.50×10^3 m for 3 significant figures
  - Use 1.500×10^3 m for 4 significant figures
  - On the safe side, we usually treat the number as 1.5×10^3 for 2 significant figures if there is no further information

Operations with Significant Figures – Multiplying or Dividing

- When **multiplying or dividing**, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the **lowest number of significant figures**.
- Example: 25.57 m x 2.45 m = 62.6 m^2 (not 62.6465)
  - This is because 2.45 m limits our result to 3 significant figures

Operations with Significant Figures – Adding or Subtracting

- When adding or subtracting, the number of decimal places in the result should equal the smallest number of decimal places in any term in the sum.
- Example: 135 cm + 3.251 cm = 138 cm (not 138.251 as 135 has no decimal place)
  - The 135 cm limits your answer to the units decimal value

Operations With Significant Figures – Summary

- The rule for addition and subtraction are different than the rule for multiplication and division
- For adding and subtracting, the ***number of decimal places*** is the important consideration, eg, 101.3 + 3.624 = 104.9 (not 104.924 because 101.3 has 1 decimal place only), 123+5.35 = 128 (not 128.35 because 123 has no decimal place),
  1.0001 + 0.0003 = 1.0004, 1.002-0.998 = 0.004 (1 significant digit).
- For multiplying and dividing, the ***number of significant figures*** is the important consideration,
  eg 8912.458 x 0.72 = 6400 (not 6416.96976 because 0.72 has 2 significant figures),
  98.76/1.21 = 81.6 (not 81.6198347 because 1.21 has 3 significant figures ...)

Rounding

- Rounding can worsen the accuracy of calculation (the more rounded figures you use the worse is the accuracy) but why are we doing it. One of the reasons is that computation can be faster with less number of digits. So we have to come up with a scheme to manage the inaccuracy, i.e., to make the inaccuracy not too inaccurate. Our scheme is as follows:

1. Last retained digit is increased by 1 if the last digit dropped is greater than 5, eg: round 99.4**7** will give 99.5.
2. Last retained digit remains as it is if the last digit dropped is less than 5, eg: round 99.4**2** will give 99.4.
3. If the last digit dropped is equal to 5, the retained digit should be rounded to the nearest even number, eg: round 99.4**5** will give 99.4, and round 99.5**5** will give 99.6. We hope by doing so in the long run we can reduce the accumulation of arithmetic inaccuracy (the terms level out by themselves in the long arithmetic process).
4. Saving rounding until the final result will help eliminate accumulation of errors, i.e., don't do rounding during the intermediate steps.

This is the procedure we use and is different from what we have learnt in secondary school.

Notes:

- What is taught here is a **proposed** model.
- This does not mean that all calculations will have to adopt the new model. For example, the Microsoft Excel Software does not adopt the proposed new model.
- When the accuracy of data is not mentioned, we assume that the data is absolutely and infinitely accurate thus the rules mentioned in this chapter do not apply. E.g., 12.34 is treated as 12.3400000000...
- We apply the rules only when its application is instructed, i.e., when you are told to do so.
- In any models, rounding should be done only in the last step. All intermediate steps and all intermediate answers should not be rounded and all digits must be kept.