PC1221 Fundamentals of Physics I

Lectures 11 and 12

Circular Motion

and

Other Applications of Newton's Laws

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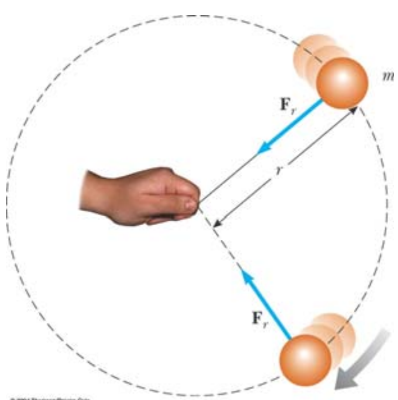
Ground Rules

- Switch off your handphone and pager
- Switch off your laptop computer and keep it
- No talking while lecture is going on
- No gossiping while the lecture is going on
- Raise your hand if you have question to ask
- Be on time for lecture
- Be on time to come back from the recess break to continue the lecture
- Bring your lecturenotes to lecture

Uniform Circular Motion (on Horizon Plan)

- A force, $\mathbf{F}_r$, is directed toward the center of the circle
- This force is associated with an acceleration, $\mathbf{a}_c$
- Applying Newton's Second Law along the radial direction gives

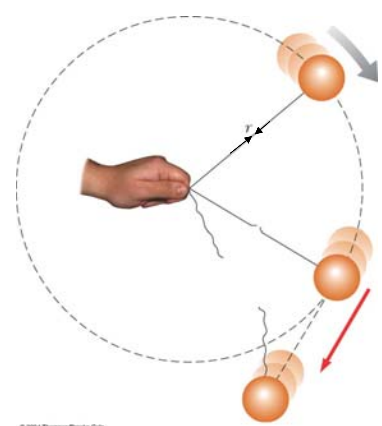
$$\sum F = ma_c = m\frac{v^2}{r}$$



Horizontal Plan

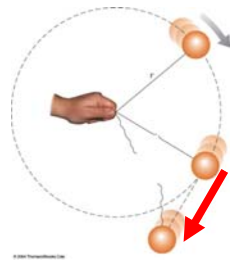
Uniform Circular Motion, cont

- A force (from my hand) causes a centripetal acceleration to act toward the center of the circle
- It causes a change in the direction of the velocity vector
- If the force vanishes, the object would move in a straight-line path tangent to the circle



Centripetal Force

- The force causing the centripetal acceleration is called the ***centripetal force***
- This is not a new force, but it is a new *role* for a force
- **Centripetal force** causes circular motion

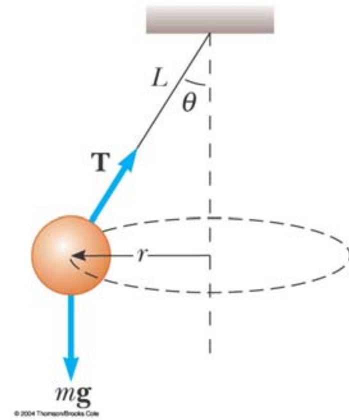


Conical Pendulum

- The object is in equilibrium in the vertical direction and undergoes uniform circular motion in the horizontal direction

$$v = \sqrt{Lg\sin\theta\tan\theta}$$

- v is independent of m
- This formula can be derived



Conical Pendulum

Centripetal force $= ma = \frac{mv^2}{r} = T\sin\theta$ — (1)

$T\cos\theta = mg$

$\Rightarrow T = \frac{mg}{\cos\theta}$ — (2)

Sub (2) into (1)

$\frac{mv^2}{r} = \frac{mg}{\cos\theta}\sin\theta$

$\Rightarrow \frac{v^2}{r} = g\tan\theta$ — (3)

But $\sin\theta = \frac{r}{L}$

$\Rightarrow r = L\sin\theta$ — (4)

Sub (4) into (3)

$\frac{v^2}{L\sin\theta} = g\tan\theta$

$\Rightarrow v^2 = Lg\sin\theta\tan\theta$

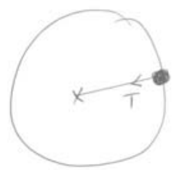
$v = \sqrt{Lg\sin\theta\tan\theta}$ #

Motion in a Horizontal Circle

- The speed at which the object moves depends on the mass of the object and the tension in the cord
- The centripetal force is supplied by the tension

$$v = \sqrt{\frac{Tr}{m}}$$

Motion in a Horizontal Circle



$T = \frac{mv^2}{r} \Rightarrow Tr = mv^2$

$v^2 = \frac{Tr}{m}$

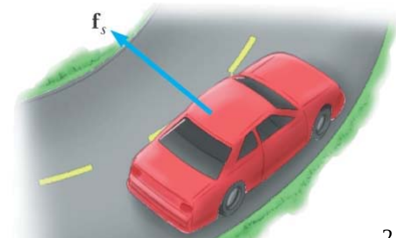
$\Rightarrow v = \sqrt{\frac{Tr}{m}}$ #

Horizontal (Flat) Curve

- The force of static friction supplies the centripetal force
- The maximum speed at which the car can negotiate the curve is

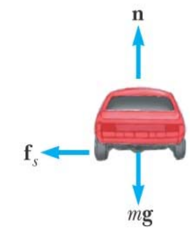
$$v = \sqrt{\mu g r}$$

- Note, the maximum velocity does not depend on the mass of the car



$$\mu mg = \frac{mv^2}{r}$$

(a)



(b)

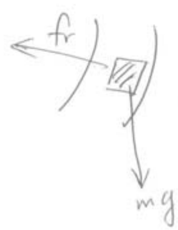
Maximum Velocity on a flat race track

fr: friction

$fr = \mu mg = \frac{mv^2}{r}$

$\Rightarrow \mu gr = v^2$

so $v = \sqrt{\mu g r}$ #



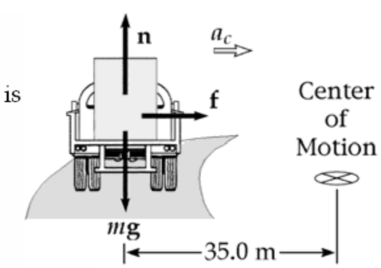
Example. A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.6, how fast can the truck be moving without the crate sliding?

Answer:

$n = mg$ since $a_y = 0$

The force causing the centripetal acceleration is the frictional force f.

From Newton's second law $f = m a_c = \frac{m v^2}{r}$.



But the friction condition is $f \le \mu_s n$ i.e., $\frac{m v^2}{r} \le \mu_s m g$

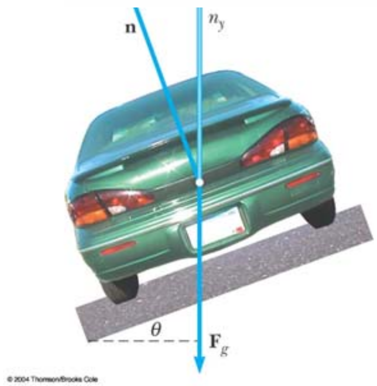
$v \le \sqrt{\mu_s r g} = \sqrt{0.600(35.0\ \mathrm{m})(9.80\ \mathrm{m/s^2})}$

$v \le \boxed{14.3\ \mathrm{m/s}}$

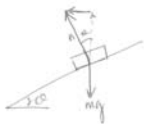
Banked Curve

- Banked curves are designed to deal with very small or no friction situation
- There is a component of the normal force that supplies the centripetal force

$$\tan\theta = \frac{v^2}{rg}$$



Speed on a Banked Curve



Vertically : $n\cos\theta = mg$

$\Rightarrow n = \frac{mg}{\cos\theta}$ — (1)

Horizontally $n\sin\theta$ = centripetal force

$= \frac{mv^2}{r}$ — (2)

sub (1) into (2) $\frac{mg}{\cos\theta}\sin\theta = \frac{mv^2}{r}$

$g\tan\theta = \frac{v^2}{r}$

$\Rightarrow \tan\theta = \frac{v^2}{rg}$ #

So $v = \sqrt{rg\tan\theta}$

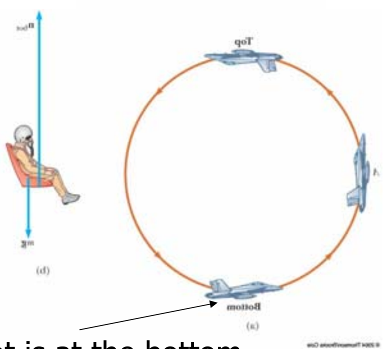
What if $v < \sqrt{rg\tan\theta}$?

What if $v > \sqrt{rg\tan\theta}$?

Loop-the-Loop in Vertical Plan

- This is an example of a vertical circle
- At the bottom of the loop (b), the upward force experienced by the object is greater than its weight

$$n_{bot} = mg\left(1 + \frac{v^2}{rg}\right)$$



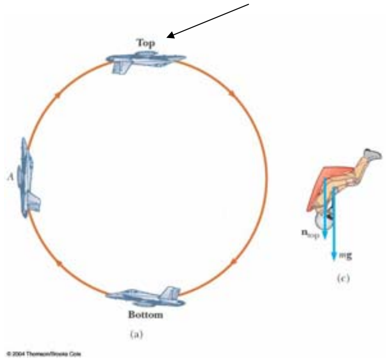
Pilot is at the bottom of the loop

Loop-the-Loop, Part 2

- At the top of the circle, the force exerted on the object is less than its weight

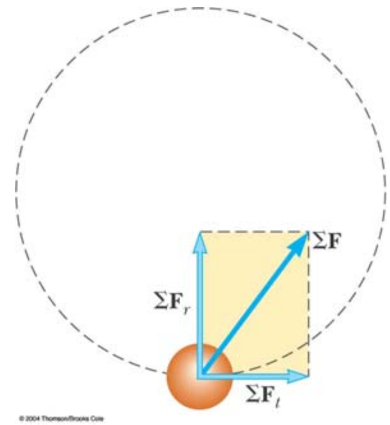
$$n_{top} = mg\left(\frac{v^2}{rg} - 1\right)$$

Pilot is at the top of the loop



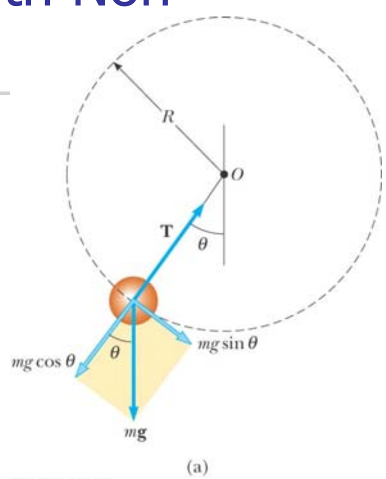
Non-Uniform Circular Motion

- The acceleration and force have tangential components
- $\mathbf{F}_r$ produces the centripetal acceleration
- $\mathbf{F}_t$ produces the tangential acceleration
- $\Sigma\mathbf{F} = \Sigma\mathbf{F}_r + \Sigma\mathbf{F}_t$



Vertical Circle with Non-Uniform Speed

- The gravitational force exerts a tangential force on the object
  - Look at the components of F_g
- The tension at any point can be found



$$T = m\left(\frac{v^2}{R} + g\cos\theta\right)$$

This formula corresponds to the previous 2 special cases when Θ= 0° and 180°.

Loop-the-Loop in Vertical Plan

(1) Bottom

$n - mg = \frac{mv^2}{r}$

$n = mg + \frac{mv^2}{r}$

$= mg\left(1 + \frac{v^2}{rg}\right)$

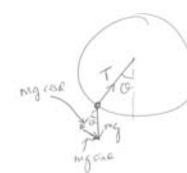
(2) Top

$n + mg = \frac{mv^2}{r}$

$n = \frac{mv^2}{r} - mg$

$= mg\left(\frac{v^2}{rg} - 1\right)$

Vertical Circle with Non-Uniform Speed



$T - mg\cos\theta = \frac{mv^2}{r}$

$\Rightarrow T = \frac{mv^2}{r} + mg\cos\theta$

$= mg\left(\frac{v^2}{rg} + \cos\theta\right)$

when $\theta = 0$ (Bottom) $T = mg\left(\frac{v^2}{rg} + 1\right)$

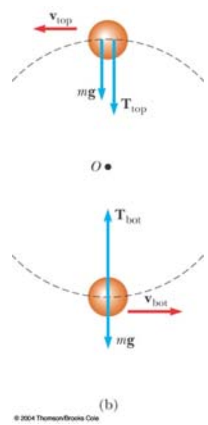
when $\theta = 180$ (Top) $T = mg\left(\frac{v^2}{rg} - 1\right)$

Top and Bottom of Circle

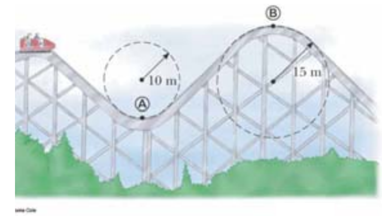
- The tension at the bottom is a maximum
- The tension at the top is a minimum
- If T_{top} = 0, then

$$v_{top} = \sqrt{gR}$$

What if the velocity is less than this value?



Examples. A roller coaster car has a mass of 500 kg when fully loaded with passengers. (a) If the vehicle has a speed of 20.0 m/s at point A, what is the force exerted by the track on the car at this point? (b) What is the maximum speed the vehicle can have at B and still remain on the track?

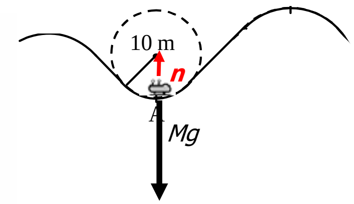


Answer

(a) $v = 20.0 \text{ m/s}$,

$n =$ force of track on roller coaster, and

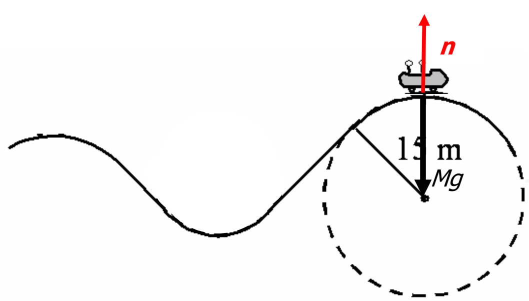
$R = 10.0 \text{ m}$.



$$\sum F = \frac{Mv^2}{R} = n - Mg$$

$$n = Mg + \frac{Mv^2}{R} = (500 \text{ kg})(9.80 \text{ m/s}^2) + \frac{(500 \text{ kg})(20.0 \text{ m/s}^2)^2}{10.0 \text{ m}}$$

$$n = 4\,900 \text{ N} + 20\,000 \text{ N} = \boxed{2.49 \times 10^4 \text{ N}}$$



(b) At B, $Mg - n = \frac{Mv^2}{R}$

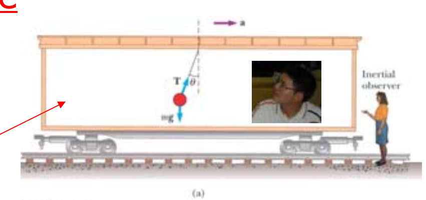
The max speed at B corresponds to

$n = 0$

$$Mg = \frac{Mv_{max}^2}{R} \Rightarrow v_{max} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1 \text{ m/s}}$$

Motion in Accelerated Frames

- A ***fictitious force*** results from an accelerated frame of reference
  - A fictitious force appears to act on an (first) object in the same way as a real force, but you cannot identify a second object that applies the fictitious force to the first object. Therefore a fictitious force is not a real force that exhibits real effects.



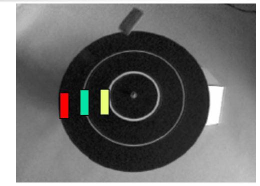
What if the observer is in the train?

"Centrifugal" Force

- From the frame of the passenger (b), a force appears to push her toward the door
- From the frame of the Earth, the car applies a leftward force on the passenger. This is the centripetal force
- The outward force is often called a ***centrifugal*** force
  - It is a fictitious force due to the acceleration associated with the car's change in direction

Which eraser will fly off first?

Three erasers are placed on a rotating disc at different distances from center and we spin the disc at increasing speed. Which eraser will fly off first (inner, middle, outer)? Why?

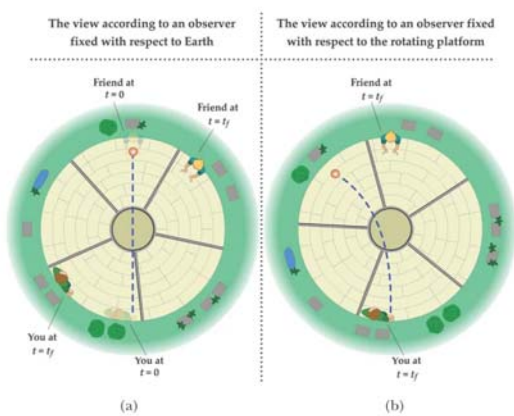


Answer:

All three erasers experience the same friction on the disc surface. The centripetal force on each eraser is different: $m\, v^2/r = m(r\omega)^2/r = mr\omega^2$, where ω is the angular velocity. As the outer eraser (with the largest r) experiences the largest centripetal force, its fictitious outward force (centrifugal force) is the largest so it will fly off first.

"Coriolis Force"

- This is an apparent force caused by changing the radial position of an object in a rotating coordinate system
- The result of the rotation is the curved path of the ball



Fictitious Forces, examples

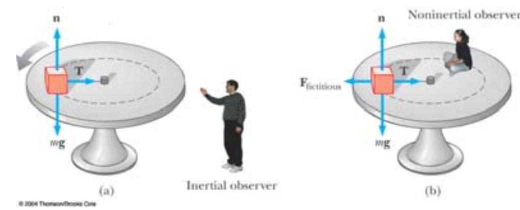
- Although fictitious forces are not real forces, they can have real effects
- Examples:
  - Objects in the car do slide
  - You can feel pushed to the outside of a rotating platform
  - The Coriolis force is responsible for the rotation of weather systems and ocean currents. How?

Ocean Currents **(has nothing to do with tides which are caused by the pull of gravity from the Moon and the Sun):**

The Earth spins on its axis from West to East (counter-clockwise) when viewed looking down on the North Pole).

Due to the rotation of the earth, currents are deflected to the right in the northern hemisphere and to the left in the southern hemisphere by coriolis effect. The deflection leads to highs and lows of sea level that causes ocean currents.

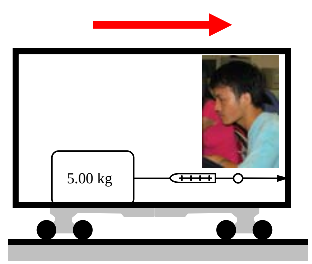
Fictitious Forces in a Rotating System



- According to the inertial observer (a), the tension is the centripetal force $T = \dfrac{mv^2}{r}$
- The noninertial observer (b) sees

$$T - F_{fictitious} = T - \frac{mv^2}{r} = 0$$

Example. An object of mass 5.00 kg, attached to a spring scale, rests on a frictionless, horizontal surface as in Figure. The spring scale, attached to the front end of a boxcar, has a constant reading of 18.0 N when the car is in motion. (a) If the spring scale reads zero when the car is at rest, determine the acceleration of the car. (b) What constant reading will the spring scale show if the car moves with constant velocity? (c) Describe the forces on the object as observed by someone in the car and by someone at rest outside the car.



Answer:

(a) $\sum F_x = Ma,\ a = \dfrac{T}{M} = \dfrac{18.0\ \mathrm{N}}{5.00\ \mathrm{kg}} = \boxed{3.60\ \mathrm{m/s^2}}$ to the right.

(b) If $v =$ const, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are (i) T and (ii) a fictitious force ($-Ma$). Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x-direction.