To be discussed on 17th September 2008 (week-6), Wednesday 8:30am to 9:30am, at S16-04-36.

Please solve the tutorial questions in advance.

1. Let $d$ be the maximum degree of concurrency in a task-dependency graph with $n$ tasks and a critical-path length $l$. Prove that

$$\left\lfloor \frac{n}{l} \right\rfloor \leq d \leq n - l + 1.$$ 

2. Given a balanced binary tree as shown.

Describe a procedure to perform all-to-all broadcast that takes time $(t_s + \frac{twmp}{2}) \log p$ for $m$-word messages on $p$ nodes. Assume that only the leaves of the tree contain nodes, and that an exchange of two $m$-word messages between any two nodes connected by bidirectional channels takes time $t_s + twmk$ if the communication channel (or a part of it) is shared by $k$ simultaneous messages.

3. Show that if the message startup time $t_s$ is zero, then the expression $twmp(\sqrt{p} - 1)$ for the time taken by all-to-all personalized communication on a $\sqrt{p} \times \sqrt{p}$ mesh is optimal within a small ($\leq 4$) constant factor.

4. Derive the cost (in terms of time) of scatter and gather operations on a linear array and 2-D mesh.
5. If a problem of size $W$ has a serial component $W_s$, prove that $W/W_s$ is an upper bound on its speedup, no matter how many processing elements are used.

6. Consider the search tree shown in (a) in which the dark node represents the solution.

   ![Diagram](image)

   (a) DFS with one processing element
   (b) DFS with two processing elements

   (i) If a sequential search of the tree is performed using the standard depth-first search (DFS) algorithm as shown in (a), how much time does it take to find the solution if traversing each arc of the tree takes one unit of time?

   (ii) Assume that the tree is partitioned between two processing elements that are assigned to do the search job, as shown in (b). If both processing elements perform a DFS on their respective halves of the tree, how much time does it take for the solution to be found? What is the speedup? Is there a speedup anomaly? If so, can you explain the anomaly?

Please solve the tutorial questions in advance to gain the maximum benefit from our tutorial session.