Real-Time Implementation of Double Frequency Modulation (DFM) Synthesis*

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A method of digital sound synthesis, double frequency modulation (DFM), is defined in terms of two modulating frequencies. Its spectral equation is derived and the harmonic spacing shown to be dependent on the ratio of the two frequencies. The real-time generation of musical sounds by DFM is demonstrated. DFM offers superior control of harmonic envelopes than simple FM, but with less computational load than asymmetrical frequency modulation (AFM).

0 INTRODUCTION

The application of frequency modulation (FM) to the synthesis of complex sounds was a major advance in musical sound synthesis [1]. In the simplest FM synthesis mode, the frequency of a sine wave (the carrier) is modulated by another sine wave (the modulator) to produce a complex waveform whose spectral characteristics depend on the parameters of the two sine waves and the degree of modulation.

The basic equation for FM is

$$x(t) = A \sin[\omega_c t + I \sin(\omega_m t)] \tag{1}$$

where

A = amplitude
ω_c = angular carrier frequency, $= 2\pi f_c$, f_c being carrier frequency
ω_m = angular modulator frequency, $= 2\pi f_m$, f_m being modulator frequency
I = modulation index
t = time.

When expressed in Fourier series form, it can be seen that the amplitude of each harmonic component of the FM spectra is governed by the Bessel functions of the first type $J_n(I)$ as follows:

$$\begin{aligned} x(t) &= A \sin[\omega_c t + I \sin(\omega_m t)] \\ &= A \sum_{n=-\infty}^{\infty} J_n(I) \sin(\omega_c + n\omega_m)t \, . \end{aligned} \tag{2}$$

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A single FM process involving one carrier and one modulator frequency is known as an FM operator. Multiple FM operators may be combined or cascaded to produce more complex waveforms and spectra. For example, the output of an FM operator may be used as the modulator frequency for a second FM operator.

1 DOUBLE FREQUENCY MODULATION

If we consider the FM Eq. (1), a special method of synthesis is obtained by modifying the equation. In this method the carrier is removed and replaced by another modulator within the square brackets. These two modulators are added, and the result is used as an argument for the overall sine function. The synthesis equation then becomes

$$x(t) = A \sin[I_1 \sin(\omega_1 t) + I_2 \sin(\omega_2 t)] \, . \tag{3}$$

We call this method of synthesis double frequency modulation (DFM).

We also note that Eq. (3) is identical to Schottstaedt's complex modulating wave [2] except that $\omega_c t$, that is, the carrier, is omitted. We can consider DFM to be a special case of Schottstaedt's FM with a complex modulating wave.

2 SPECTRAL ANALYSIS OF DFM

To obtain the spectral representation of DFM, we first make use of Eq. (1), that is, simple FM, and then treat DFM as an extension of FM with an extra modulator, but with the carrier frequency set to zero. It can be shown [3] that

$$x(t) = A \sum_i \sum_k J_i(I_1)J_k(I_2) \sin(i\omega_1 t + k\omega_2 t) . \quad (4)$$

Eq. (4) is the DFM spectral equation, which we can use as the basis for an algorithm to do spectral simulation using a digital computer.

In Eq. (4) the indices i and k run from $-\infty$ to ∞, giving rise to frequency components that lie in the negative and positive regions of the frequency axis. We have to reflect the negative frequency harmonics around 0 Hz with their phases inverted and add them to harmonics that appear in the positive region. These lines are called reflected side frequencies.

Fig. 1 [1] illustrates this. In Fig. 1(a) we have an example of a spectrum with components in their original positions. In Fig. 1(b) the harmonics on the negative side of the frequency axis are reflected around 0 Hz, inverted in phase, and added to the harmonics on the positive side of the frequency axis. For example, the -200-Hz component is inverted and added to the $+200$-Hz component, thus decreasing the energy at 200 Hz. Similarly, the -300-Hz component is added to the $+300$-Hz component after inversion. Since the inverted -300-Hz component is of the same phase as the $+300$-Hz component, the combined 300-Hz energy is increased.

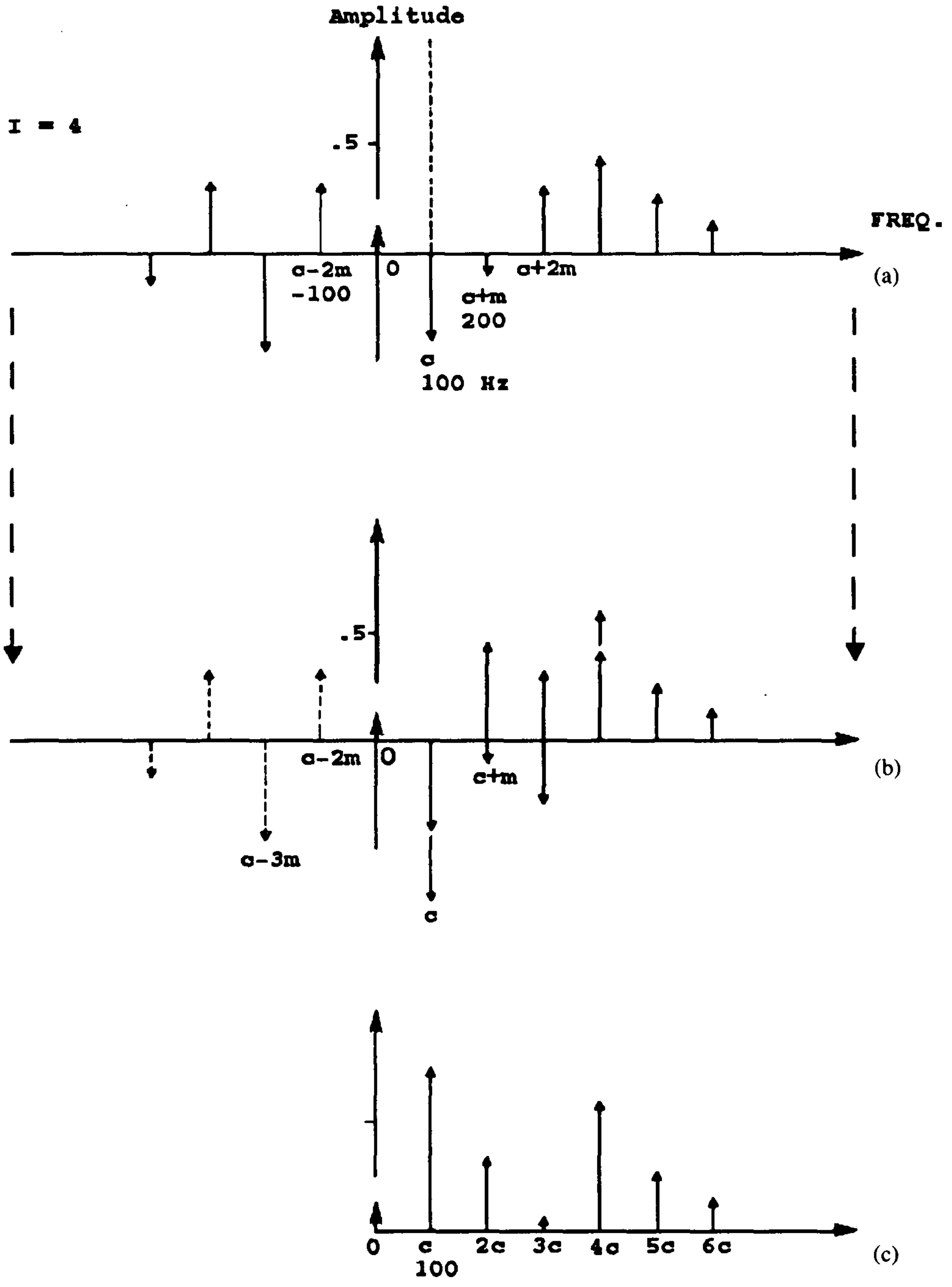

Fig. 1. Demonstration of reflected side frequencies.

3 DFM HARMONICS

We have written a program in C to generate the DFM harmonics using Eq. (4). By studying the output of the program it is evident that the frequencies of the DFM harmonics depend on the relative values of f_1 and f_2. We consider the case when f_2 is greater than f_1, and $f_2/f_1 = N_2/N_1$, where N_1 and N_2 are nonzero integers not having a common factor. The following empirical rules for the frequencies generated apply for the DFM harmonics.

3.1 N_1 Odd, N_2 Odd

We first consider the case when N_1 is an odd number. If N_2 is also an odd number, then the frequency difference between harmonics will be $2f_1/N_1$, that is, the harmonic frequencies are given by

$$f = f_1 \pm n\frac{2f_1}{N_1}, \qquad n = 0, 1, 2, 3, \cdots .$$

Fig. 2 illustrates this for $f_1 = 200$ Hz and $f_2 = 600$ Hz. In this case $N_1 =$ and $N_2 = 3$ and the spacing between successive harmonics is $2 \times 200/1$ Hz, that is, 400 Hz. Fig. 3 shows the case when $f_1 = 200$ Hz and $f_2 = 1000$ Hz, that is, N_1 and $N_2 = 5$. The harmonic spacing is still 400 Hz, but the relative amplitudes of the harmonics are different.

Figs. 4 and 5 also illustrate this for the case $N_1 = 3$. In Fig. 4 $f_1 = 300$ Hz and $f_2 = 500$ Hz, that is, $N_2 = 5$; in Fig. 5 $f_1 = 300$ Hz and $f_2 = 700$ Hz, that is, $N_2 = 7$. These two cases have the same harmonic spacing of $(2 \times 300)/3$ Hz, that is, 200 Hz, but the relative amplitudes of the harmonics are different.

3.2 N_1 Odd, N_2 Even

For the case when N_1 is odd and N_2 is even, the frequency difference between harmonics will be f_1/N_1, that is, the harmonic frequencies are given by

$$f = f_1 \pm \frac{nf_1}{N_1} \qquad n = 0, 1, 2, 3, \cdots .$$

Figs. 6 and 7 give examples for $f_1 = 200$ Hz and $N_1 = 1$, for $N_2 = 2$ and 4, respectively, that is, $f_2 = 400$

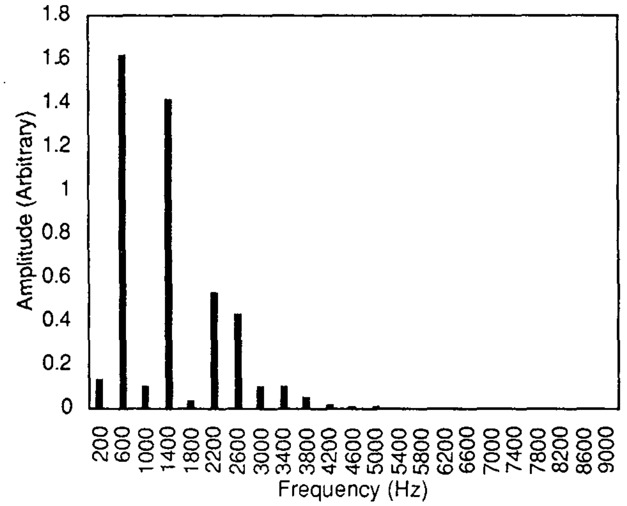

Fig. 2. $f_1 = 200$ Hz, $f_2 = 600$ Hz, $I_1 = I_2 = 3.0$.

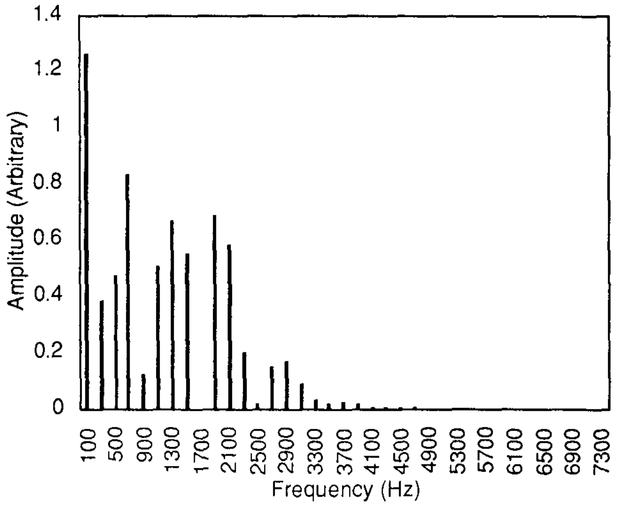

Fig. 4. $f_1 = 300$ Hz, $f_2 = 500$ Hz, $I_1 = I_2 = 3.0$.

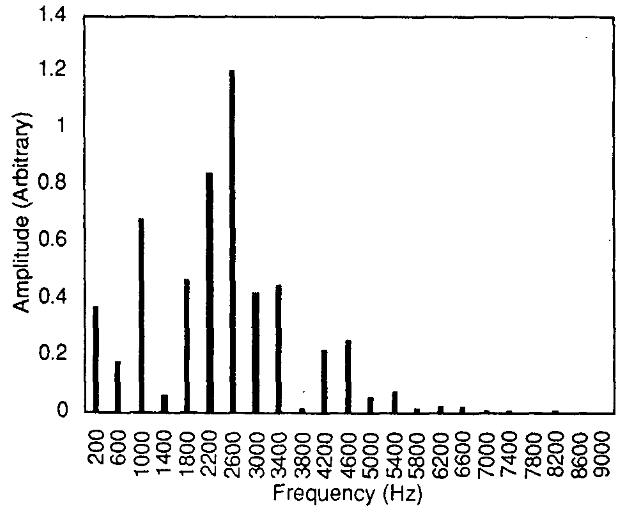

Fig. 3. $f_1 = 200$ Hz, $f_2 = 1000$ Hz, $I_1 = I_2 = 3.0$.

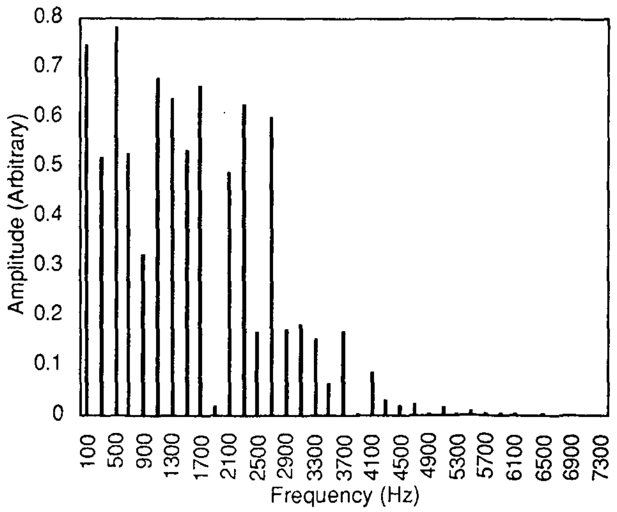

Fig. 5. $f_1 = 300$ Hz, $f_2 = 700$ Hz, $I_1 = I_2 = 3.0$.

and 800 Hz. The spacing between harmonics is 200/1, that is, 200 Hz. The relative amplitudes of the harmonics vary with the actual values of N_2 and hence f_2.

Figs. 8 and 9 give examples for $f_1 = 300$ Hz and $N_1 = 3$, for $N_2 = 4$ and 8, respectively, that is, $f_2 = 400$ and 800 Hz. The spacing between harmonics is 300/3, that is, 100 Hz. Again, the relative amplitudes of the harmonics vary with the actual values of N_2 and hence f_2.

3.3 N_1 Even, N_2 Odd

For even values of N_1, N_2 will always be an odd number. In this case the frequency difference between harmonics will also be f_1/N_1, that is, the harmonics frequencies are also given by

$$f = f_1 \pm \frac{nf_1}{N_1}, \qquad n = 0, 1, 2, 3, \cdots .$$

Figs. 10–12 give examples for $f_1 = 200$ Hz and $N_1 = 2$, for $N_2 = 3$, 5, and 7, respectively, that is, $f_1 = 300$, 500, and 700 Hz. In all three cases the spacing between harmonics is 200/2, that is, 100 Hz. However, the relative amplitudes of the harmonics vary with the values of N_2 and hence f_2. It may be observed from Figs. 10–12 that many of the harmonics are diminished in amplitude for higher values of N_2.

3.4 f_1 or f_2 Is Zero

When either one of the two frequencies is absent, for example, f_2 is zero, then Eq. (3) will contain only f_1 or ω_1,

$$x(t) = A \sin[I_1 \sin(\omega_1 t)] .$$

Then the harmonics generated are given by

$$f = f \pm nf, \qquad n = 0, 1, 2, 3, \cdots .$$

The spacing between harmonics is equal to f_1. This can also be interpreted as a case of $N_1 = 1$ and N_2 even, regarding zero as an even integer. It can also be regarded as a special case of simple FM, with the carrier frequency f_c equal to zero. It may be observed that some of the harmonics are insignificant though not zero.

Fig. 13 illustrates this for $f_1 = 200$ Hz and $f_2 = 0$ Hz. The spacing between successive harmonics is 200

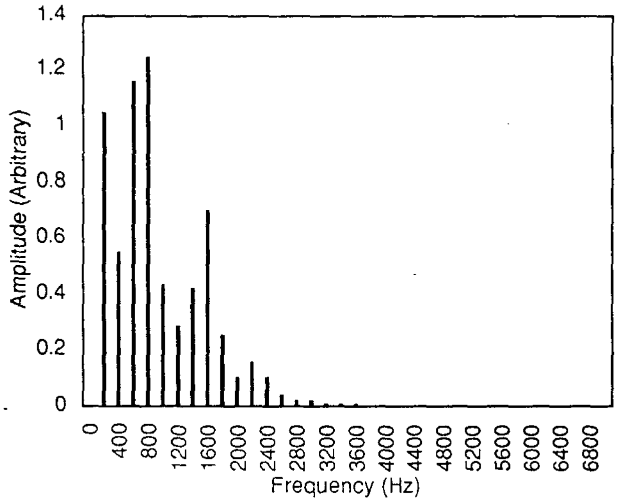

Fig. 6. $f_1 = 200$ Hz, $f_2 = 400$ Hz, $I_1 = I_2 = 3.0$.

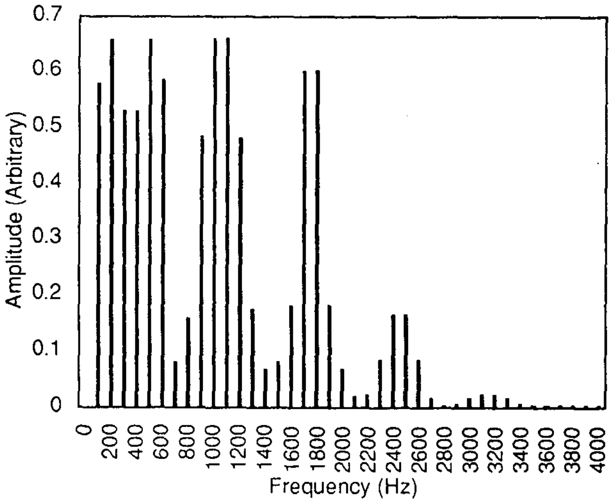

Fig. 8. $f_1 = 300$ Hz, $f_2 = 400$ Hz, $I_1 = I_2 = 3.0$.

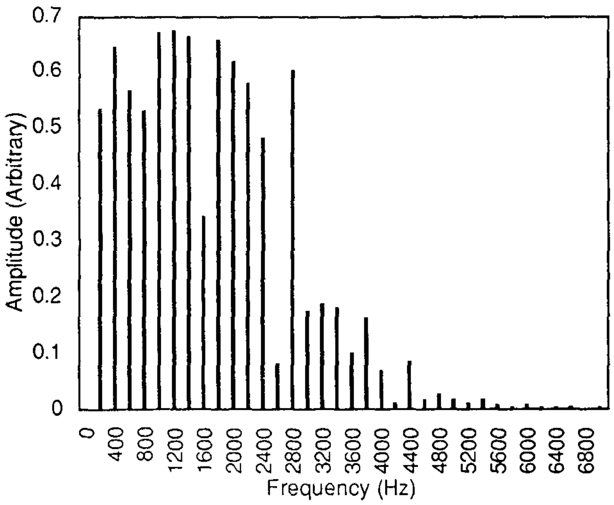

Fig. 7. $f_1 = 200$ Hz, $f_2 = 800$ Hz, $I_1 = I_2 = 3.0$.

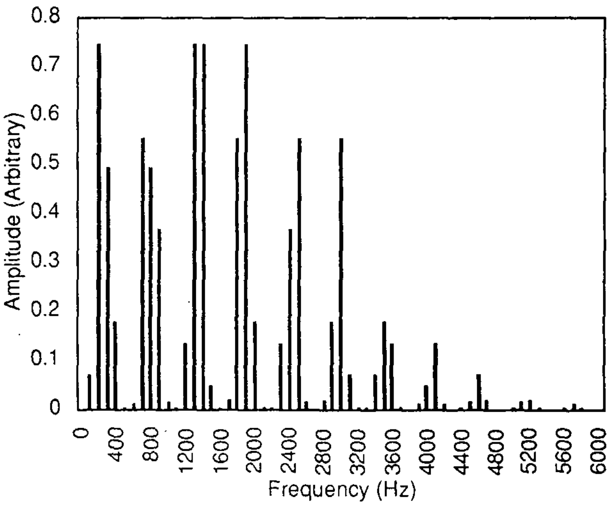

Fig. 9. $f_1 = 300$ Hz, $f_2 = 800$ Hz, $I_1 = I_2 = 3.0$.

Hz. However, the 400; 800; 1200-Hz frequency components have very small but nonzero amplitudes that are negligible when compared to the 200- and 600-Hz frequency components.

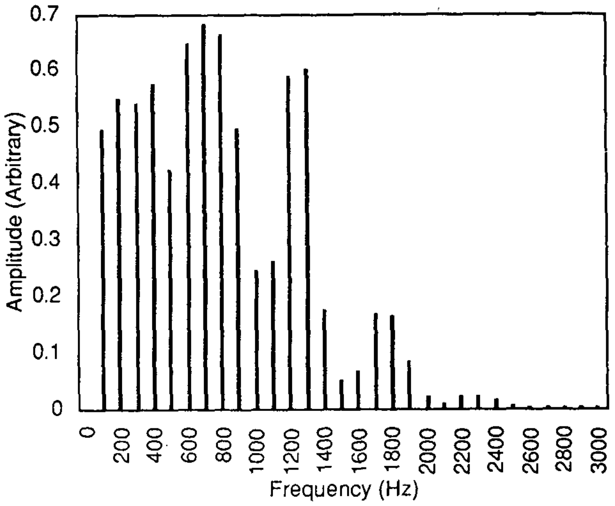

Fig. 10. f_1 = 200 Hz, f_2 = 300 Hz, $I_1 = I_2$ = 3.0.

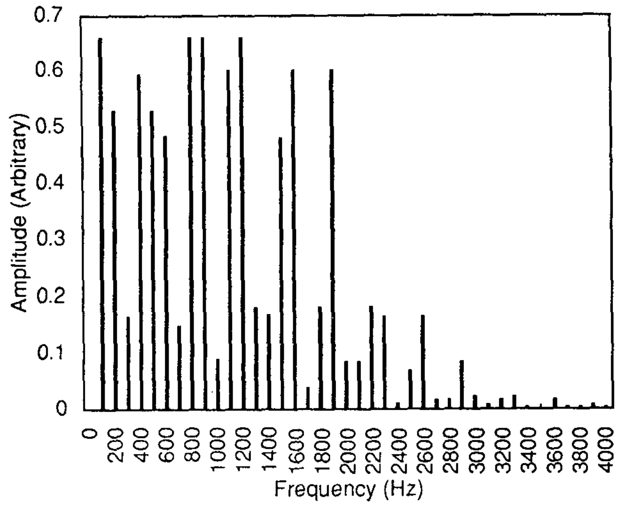

Fig. 11. f_1 = 200 Hz, f_2 = 500 Hz, $I_1 = I_2$ = 3.0.

4 ADVANTAGES OF DFM

DFM thus provides an alternative method of digital sound synthesis in which, like simple FM synthesis, harmonic frequencies can be generated from two given frequencies. In FM the harmonic spacing is the difference between the carrier frequency and the modulating frequency. In DFM, for a given value of f_1 it is possible to produce harmonics with spacings of $2f_1, f_1$, or a submultiple of f_1, depending on the ratio of f_1 and f_2. The relative amplitudes of the harmonics are dependent on the actual value of the higher of the two frequencies and the indices. The special case of DFM when one frequency is zero is equivalent to the special case of FM when the carrier frequency is zero.

FM has deservedly become recognized as one of the most efficient methods of digitally generating musical sounds with rich harmonic components. However, the harmonics are limited by the fact that the FM spectrum is arranged symmetrically on either side of the carrier frequency. In order to obtain complex harmonic envelopes, it may be necessary to employ complex combinations of several FM operators. Asymmetrical FM or AFM [5] is able to overcome this by employing an additional parameter which modifies the simple FM equation. This results in FM spectra that are asymmetrical about the carrier frequency and hence able to simulate more complex spectral envelopes with fewer FM operators. The AFM method of synthesis, while practical for real-time generation [6], imposes a greater computational load on the digital generator compared to simple FM.

DFM synthesis offers an alternative to FM and AFM in that sounds with more complex harmonic envelopes than simple FM may be generated, while avoiding the computational load of AFM. In DFM we have only three sine functions, two multiplications, and one addition. In AFM there are two sine functions, one cosine function, one exponential function, five multiplications, two divisions, two additions, and one subtraction, which is computationally much heavier than DFM. For example, we

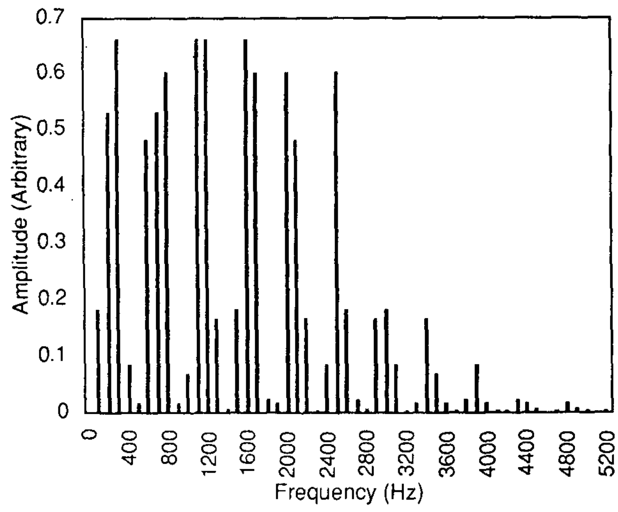

Fig. 12. f_1 = 200 Hz, f_2 = 700 Hz, $I_1 = I_2$ = 3.0.

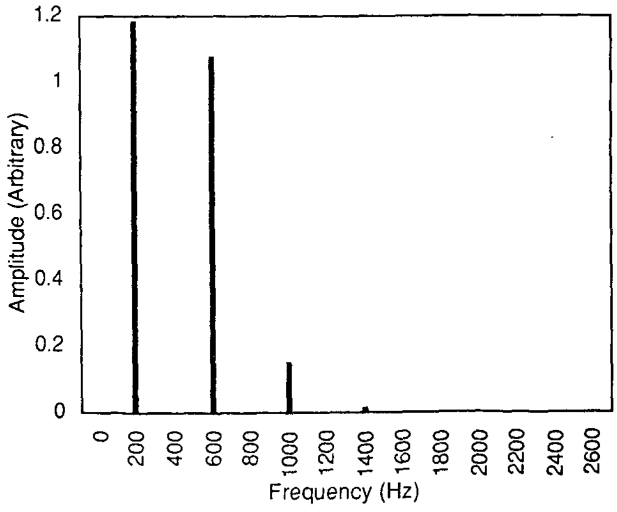

Fig. 13. f_1 = 200 Hz, f_2 = 0 Hz, I_1 = 3.0, I_2 = 0.0.

have found that DFM is able to control the symmetry of the spectrum around a dominant spectrum line to a certain extent, depending on both f_1 and f_2.

Let us illustrate this by giving an example. In Fig. 14 we have $f_1 = 200$ Hz, $I_1 = 1.3$, and $f_2 = 800$ Hz while I_2 runs from 1.6 to 2.4 in steps of 0.2. In this figure, as I_2 increases, the power drifts from the left side of the dominant harmonic (800 Hz) to the right side.

When the ratio of f_1 and f_2 is high and I_1 is smaller than I_2—for example, when $f_1 = 200$ Hz, $f_2 = 2000$ Hz, $I_1 = 0.8$, and $I_2 = 7.7$—the spectrum of DFM has a special characteristic. In Fig. 15 we can see that there

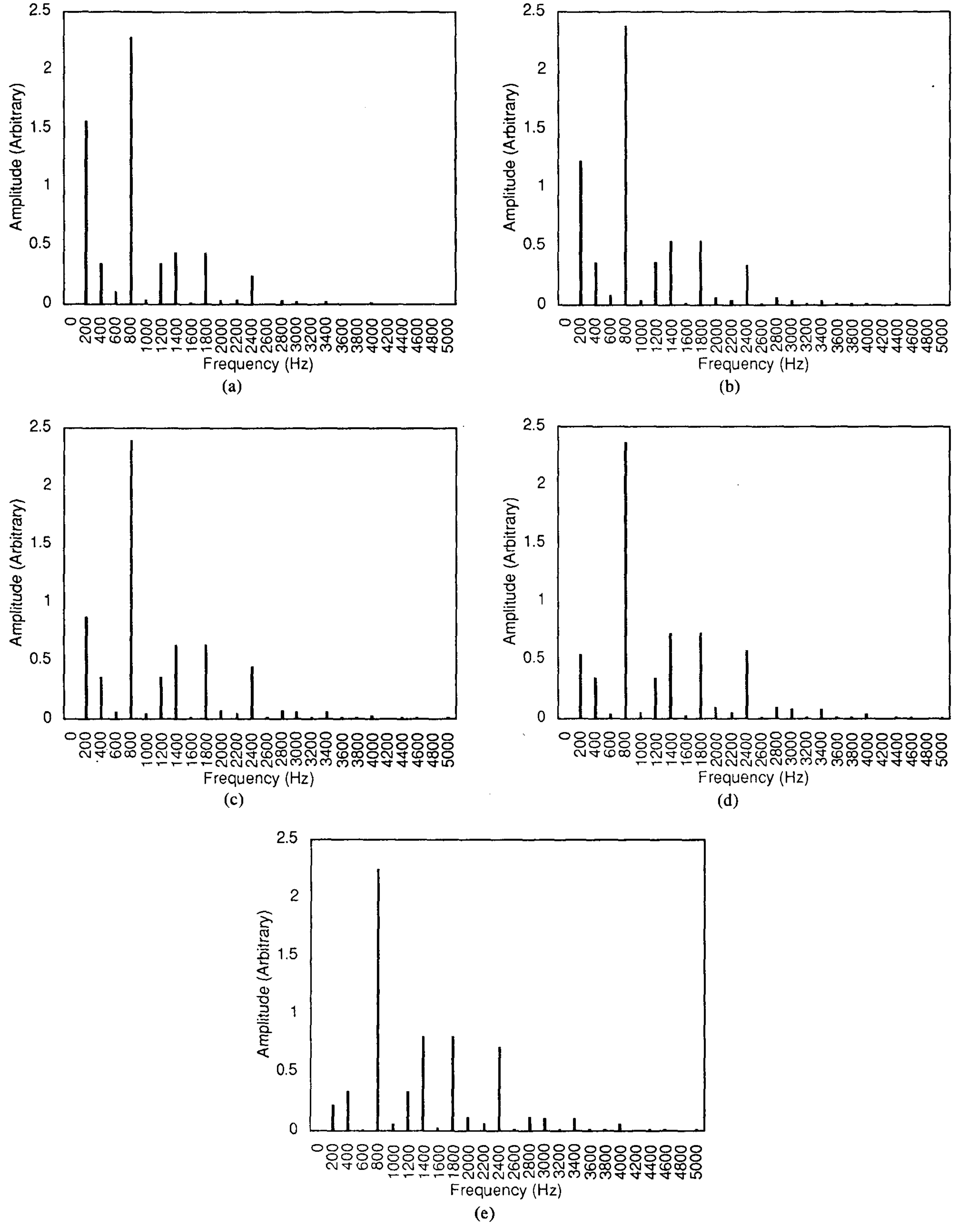

Fig. 14. $f_1 = 200$ Hz, $f_2 = 800$ Hz, $I_1 = 1.3$. (a) $I_2 = 1.6$. (b) $I_2 = 1.8$. (c) $I_2 = 2.0$. (d) $I_2 = 2.2$ (e) $I_2 = 2.4$.

are several clusters of harmonics, and each cluster has either two harmonics of the same amplitude or one dominant harmonic with two sidebands of equal amplitudes. When I_2 is smaller, we will get fewer of these clusters. We also have observed that when I_2 is increased from a lower value to a higher value, more and more of these clusters will be generated at higher frequencies.

5 DFM SYNTHESIS OF MUSICAL INSTRUMENTS

To synthesize the waveform of a musical instrument, the spectrum of the instrument to be synthesized has to be obtained. If its waveform is available, either from real-time performance or from a recording, its spectrum may be obtained by using an FFT spectrum analyzer to perform a spectrum analysis. From an inspection of the spectrum, the DFM parameters such as f_1, f_2, I_1, and I_2 may be chosen for which the synthesized DFM waveform would have the closest spectral characteristics to that of the real sample.

We obtained waveforms of a number of musical instruments from the McGill University Master Samples Compact Discs (MUMS CDs). The MUMS CD set is a convenient source of musical instrument waveforms, as it contains sampled waveforms of real musical instruments. We used the waveforms for harpsichord, trumpet, and pipe organ from the MUMS CDs and obtained the spectra of these waveforms by subjecting them to spectral analysis in a dedicated FFT analyzer. We then used an Apple Macintosh computer equipped with a Motorola DSP56000 digital signal processor (DSP) to do the DFM synthesis and generate waveforms with spectral harmonics as close to the spectra of these musical instruments as possible. The DFM waveforms were then analyzed to compare their spectra with those of the original waveforms from the MUMS CDs.

In Figs. 16–18 the tables list the various parameter values used for the DFM synthesis of the sampled instruments. The spectrum plot after each table compares the synthesized DFM waveform spectrum with that of the

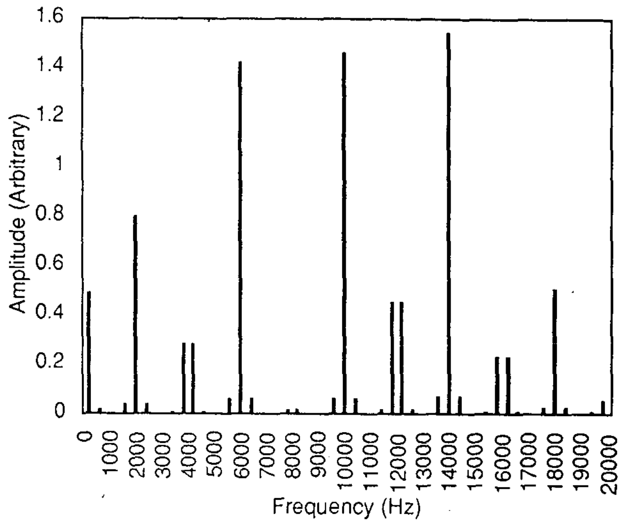

Fig. 15. f_1 = 200 Hz, f_2 = 2000 Hz, I_1 = 0.8, I_2 = 7.7.

DFM OPERATOR A			DFM OPERATOR B		
AmpA	=	50	AmpB	=	50
IndexA1	=	0.8	IndexB1	=	0.7
FreqA1	=	440 Hz	FreqB1	=	1320 Hz
IndexA2	=	1.9	IndexB2	=	-0.35
FreqA2	=	880 Hz	FreqB2	=	1760 Hz

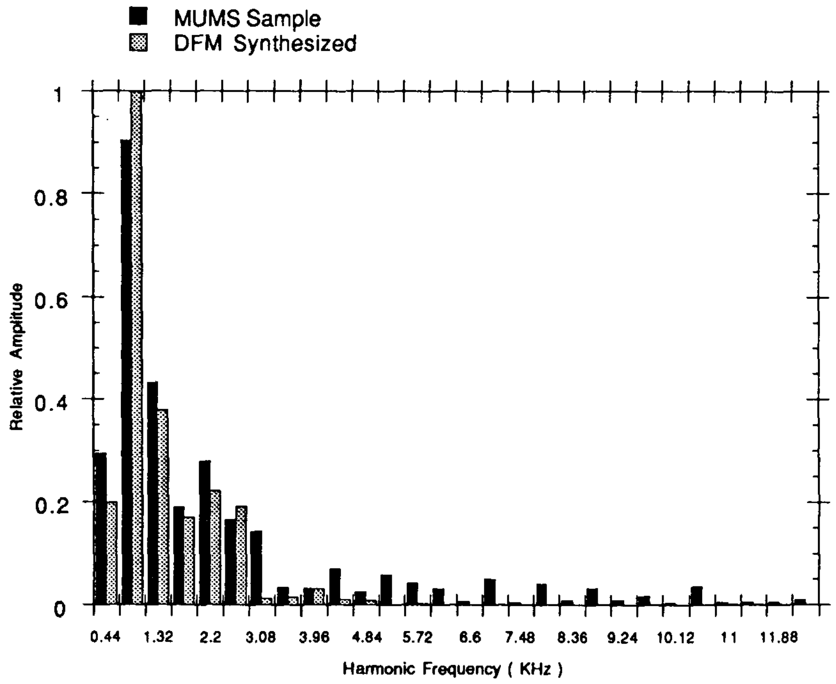

Fig. 16. Harpsichord.

DFM OPERATOR A			DFM OPERATOR B		
AmpA	=	43	AmpB	=	10
IndexA1	=	4.6	IndexB1	=	-1.55
FreqA1	=	220 Hz	FreqB1	=	660 Hz
IndexA2	=	3.95	IndexB2	=	-1.35
FreqA2	=	440 Hz	FreqB2	=	1980 Hz

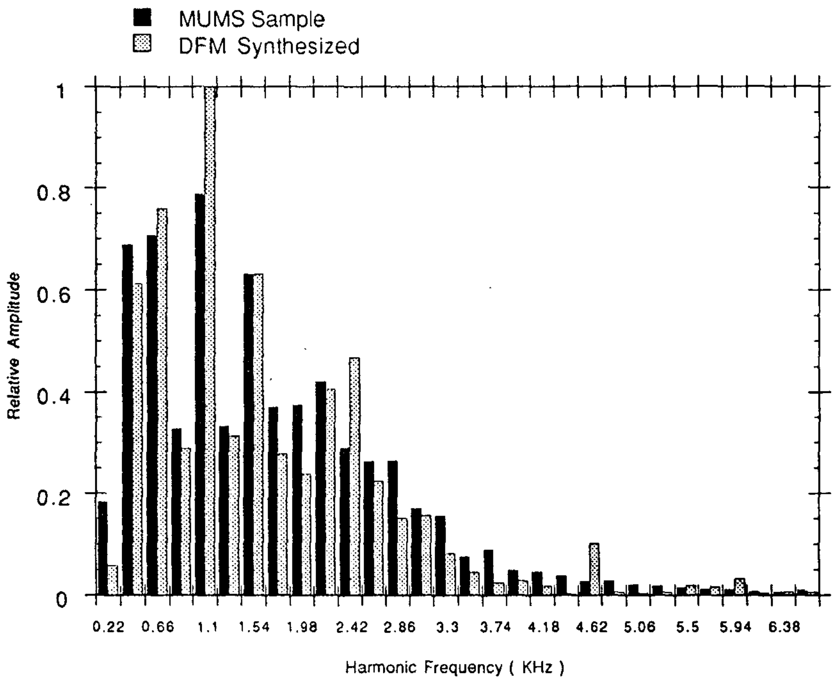

Fig. 17. Trumpet.

DFM OPERATOR A	DFM OPERATOR B	DFM OPERATOR C
AmpA = 60	AmpB = 42	AmpC = 15
IndexA1 = 1.9	IndexB1 = 0.81	IndexC1 = 0.68
FreqA1 = 440 Hz	FreqB1 = 2620 Hz	FreqC1 = 5280 Hz
IndexA2 = 1.58	IndexB2 = 0.68	IndexC2 = 0.9
FreqA2 = 880 Hz	FreqB2 = 3500 Hz	FreqC2 = 7040 Hz

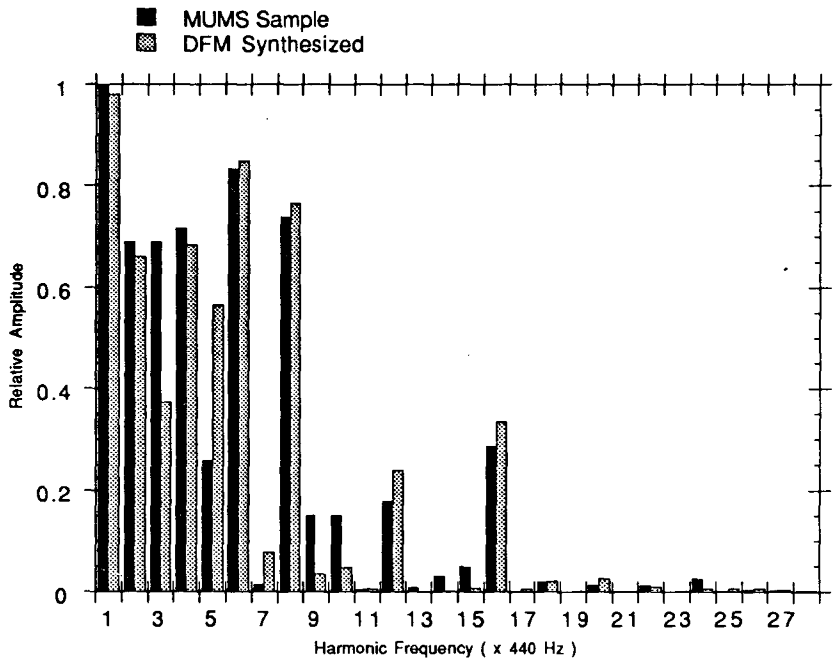

Fig. 18. Pipe organ.

original sampled waveform from the MUMS CDs. We also note that the spectrum being modeled is a steady-state spectrum.

6 CONCLUSION

In this paper we described a special method of digital sound synthesis called double frequency modulation (DFM) and demonstrated its real-time implementation. Using two modulator frequencies f_1 and f_2, DFM is able to generate waveforms whose spectra have harmonic spacings of $2f_1$, f_1, or submultiples of f_1, depending on the ratio of f_1 and f_2. The relative amplitudes of the harmonics are not restricted to a symmetrical envelope and can be controlled by judicious selection of f_1 and f_2 and the modulation indices I_1 and I_2. DFM therefore may be a useful alternative method of synthesis to simple FM synthesis, whose harmonics always have a symmetrical envelope about the carrier frequency.

We have therefore shown that DFM synthesis can be implemented in real time on a modern DSP such as the Motorola DSP56000. We implemented two DFM operators, which contain six sine wave oscillators, to show that the DFM synthesis can synthesize musical instruments fairly accurately.

Real-time DFM synthesis thus offers an attractive method of synthesis with better control of harmonic envelopes than simple FM, while requiring a lower computational load than AFM.

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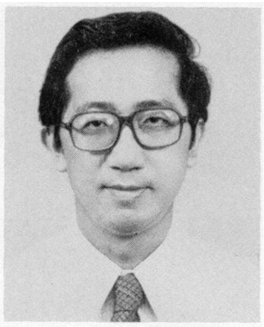

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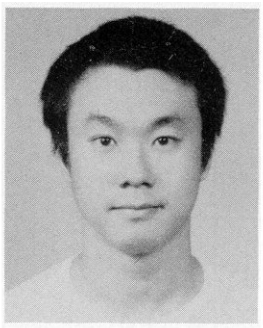

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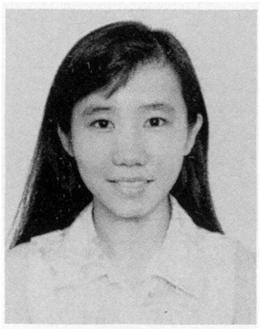

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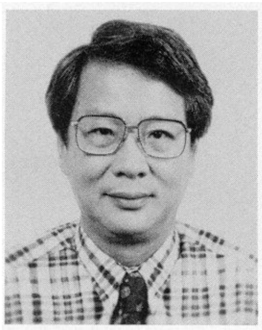

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