Question 1 (a)

Multipoles of order L transfer angular momentum of $L\hbar$ per photon. So a γ-decay from an initial nuclei with state I_i and π_i to final nuclei with state I_f and π_f, we get $\vec{I_i} = \vec{L} + \vec{I_f}$ by conservation of energy. Angular momentum coupling gives us $|I_i - I_f| \leq L \leq I_i + I_f$. $L \neq 0$ since there is no monopole radiation.

As for parity, we know that $\vec{E}$ is a polar vector, while $\vec{B}$ is an axial vector. Under parity change, $\vec{E} \to -\vec{E}$ and $\vec{B} \to \vec{B}$. So electric dipoles have odd parities and magnetic dipoles have even parities. The parities of the radiation field can be represented by the expressions:

$\pi(ML) = (-1)^{L+1}, \qquad \pi(EL) = (-1)^L$

∴ The selection rules,

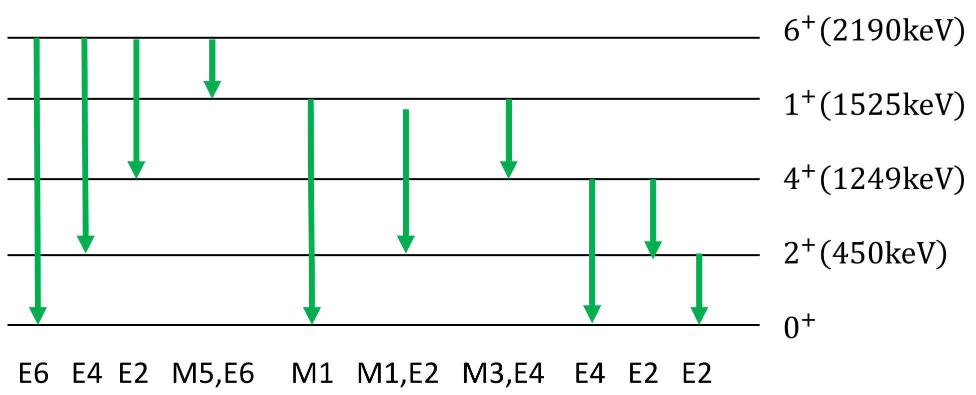
$|I_i - I_f| \leq L \leq I_i + I_f, \qquad L \neq 0$

$\Delta\pi = \text{yes} \quad \Rightarrow \quad \text{odd } M \text{ even } E$

$\Delta\pi = \text{no} \quad \Rightarrow \quad \text{odd } E \text{ even } M$

Question 1 (b) (i)

All of them have $\Delta\pi = \text{no}$, so they all have even E and odd M.



Question 1 (b) (ii)

Energies that can be ignored have very small transition probabilities: $E6, E4, M5, M3, E4$. So we would expect to see the following transitions:

$1^+ \to 0^+ \quad 1525\text{keV} \quad M1$

$1^+ \to 2^+ \quad 1075\text{keV} \quad M1, E2$

$6^+ \to 4^+ \quad 941\text{keV} \quad E2$

$4^+ \to 2^+ \quad 799\text{keV} \quad E2$

$2^+ \to 0^+ \quad 450\text{keV} \quad E2$

Question 1 (c)

$Q = m(H) + m(n) - m(^2H) = 2.22MeV/c^2$

Question 2 (a)

Cyclotron is also called the magnetic resonance accelerator. An ion beam is bent into circular path by a magnetic field, and it orbits inside 2 semi-circlular metal chambers (called 'dees'). The dees are connected to an AC voltage. These ions are accelerated each time they cross the gap, and spiral outward from the centre and is eventually extracted.

The cyclotron resonance frequency is derived using the Lorentz force in a circular orbit,

$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad v = \frac{qBr}{m}$$

The time taken for one whole circular orbit,

$$t = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \quad \Rightarrow \quad f = \frac{qB}{2\pi m}$$

Question 2 (b)

$R = 50\text{cm}, \;\; B = 1.2\text{T}$

$$E_{max} = \frac{1}{2}mv_{max}^2 = \frac{(qBR)^2}{2m} = 17.2\text{MeV}$$

Question 2 (c)

$p + {}^{15}N \rightarrow {}^{16}O^*$

The possible de-excitation pathways:

i) Gamma rays,

$p + {}^{15}N \rightarrow {}^{16}O^* \rightarrow {}^{16}O + \gamma$

$E_{ex} = m(p) + m({}^{15}N) - m({}^{16}O) = 12.13\text{MeV}$

ii) Protons,

$p + {}^{15}N \rightarrow {}^{16}O^* \rightarrow {}^{15}N + p$

$E_{ex} = 0$, since it de-excites through the same route it was created.

iii) Alpha particles,

$p + {}^{15}N \rightarrow {}^{16}O^* \rightarrow {}^{12}C + {}^{4}He$

$E_{ex} = m(p) + m({}^{15}N) - m({}^{12}C) - m({}^{4}He) = 4.97\text{MeV}$

iv) Neutrons,

$p + {}^{15}N \rightarrow {}^{16}O^* \rightarrow {}^{15}O + n$

$E_{ex} = m(p) + m({}^{15}N) - m({}^{15}O) - m(n) = -3.54\text{MeV}$

This means that energy is required for the reaction to happen instead of releasing it.

$\therefore$ This route of de-excitation is not possible.

Question 3 (a)

Mirror nuclei means that the nuclei have the same A but have proton numbers Z and $Z-1$ respectively. This means that for both nuclei, $A = Z + Z - 1 = 2Z - 1$. So the difference in the Coulomb term,

$$\begin{aligned}\Delta E_c &= a_3 Z(Z-1)A^{-\frac{1}{3}} - a_3(Z-1)(Z-2)A^{-\frac{1}{3}} \\ &= a_3(2Z-2)A^{-\frac{1}{3}} \\ &= a_3(A-1)A^{-\frac{1}{3}} \\ &= a_3\left(A^{\frac{2}{3}} - A^{-\frac{1}{3}}\right)\end{aligned}$$

Question 3 (b)

In a beta decay, $p \to n + e^+ + \nu_e$. Assuming that ν_e is massless, we have by conservation of energy, $m_p c^2 = m_n c^2 + T_n + m_e c^2 + T_e$. For the electron to have maximum kinetic energy, $T_n = 0$ (neutron at rest). Setting $c = 1$,

$$m_p = m_n + m_e + E_{max}$$

$$m_p - m_n = m_e + E_{max}$$

$$m(A, Z+1) - m(A, Z) = m_e + E_{max} \quad \text{[shown]}$$

Question 3 (c)

$$m(A,Z) = Zm_p + Nm_n - \frac{B(A,Z)}{c^2} = Zm_p + (Z+1)m_n + \frac{B(A,Z)}{c^2}$$

Setting $c = 1$,

$$\begin{aligned}m(A,Z+1) - m(A,Z) &= (Z+1-Z)m_p + (Z-Z-1)m_n + B(A,Z) - B(A,Z+1) \\ &= m_p - m_n - a_3\left(A^{\frac{2}{3}} - A^{-\frac{1}{3}}\right) \\ &= m_e + E_{max}\end{aligned}$$

$$m_e + E_{max} - m_n - m_p = a_3\left(\frac{R^2}{R_0^2} - \frac{R_0}{R}\right)$$

In MeV's,

$$0.511 + 1.29 + E_{max} = 0.72\left(\frac{R^2}{1.51 \times 10^{-30}} - \underbrace{\frac{1.23 \times 10^{-15}}{R}}_{\approx 0,\ \text{negligible}}\right)$$

$$R = \sqrt{\frac{1.80 + E_{max}}{4.76 \times 10^{29}}}$$

$$\therefore R({}^{11}_{6}C) = 180.1\text{pm}, \quad R({}^{23}_{12}Mg) = 215.3\text{pm}, \quad R({}^{39}_{20}Ca) = 248.4\text{pm}$$

Question 3 (d)

$\frac{5^+}{2} \to \frac{1^-}{2}, \quad \Delta I = 2; \Delta\pi = \text{yes}, \quad \Rightarrow \quad \text{First forbidden}$

$2^+ \to 0^+, \quad \Delta I = 2; \Delta\pi = \text{no}, \quad \Rightarrow \quad \text{Second forbidden}$

$0^+ \to 2^+, \quad \Delta I = 2; \Delta\pi = \text{no}, \quad \Rightarrow \quad \text{Second forbidden}$

$\frac{1^+}{2} \to \frac{1^-}{2}, \quad \Delta I = 0,1; \Delta\pi = \text{yes}, \quad \Rightarrow \quad \text{First forbidden}$

Question 4 (a)

a) Allowed. Electromagnetic interaction.
b) Forbidden. Baryon number not conserved.
c) Forbidden. Electron lepton number not conserved.
d) Allowed. Weak interaction.
e) Allowed. Strong interaction.
f) Allowed. Strong interaction (fusion process).
g) Allowed. Strong interaction.
h) Allowed. Weak interaction (strangeness not conserved).

Question 4 (b)

The beta decay experiment done in 1930's had a problem. It doesn't have continuous electron energy, and the angular momentum coupling law is violated. Wolfgang Pauli proposed that there should be a 3rd particle. Erico Fermi developed a comprehensive theory of beta decay, which is able to produce a continuous spectrum of the electron energies. Then finally in 1956, the neutrino is discovered by Clyde Cowen and Frederick Reines in the "Cowan-Reines neutrino experiment". They won a Nobel prize in 1995! :D

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