PC2174

Tutorial 3: Fourier Series

1. Prove the following orthogonality relations:

   \[ \int_{x_0}^{x_0+L} \sin \left( \frac{2\pi rx}{L} \right) \cos \left( \frac{2\pi px}{L} \right) \, dx = 0 \]

   \[ \int_{x_0}^{x_0+L} \cos \left( \frac{2\pi rx}{L} \right) \cos \left( \frac{2\pi px}{L} \right) \, dx = \begin{cases} L, & r = p = 0 \\ \frac{1}{2}L, & r = p > 0 \\ 0, & r \neq p \end{cases} \]

   \[ \int_{x_0}^{x_0+L} \sin \left( \frac{2\pi rx}{L} \right) \sin \left( \frac{2\pi px}{L} \right) \, dx = \begin{cases} 0, & r = p = 0 \\ \frac{1}{2}L, & r = p > 0 \\ 0, & r \neq p \end{cases} \]

2. Find the Fourier series of the functions \( f(x) = x \) in the range \(-\pi < \ x < \pi \). Hence show that

   \[ 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4} \]

3. Find the Fourier coefficients in the expansion if \( f(x) = \exp x \) over the range \(-1 < x < 1\). What value will the expansion have when \( x = 2 \)?

4. By integrating term by term the Fourier series found in the previous question, show that \( \int \exp x \, dx = \exp x + c \). Why is it not possible to show that \( d(\exp x)/dx = \exp x \) by differentiating the Fourier series of \( f(x) = \exp x \) in a similar manner?

5. Express the function \( f(x) = x^2 \) as a Fourier sine series in the range \( 0 < x \leq 2 \), and show that it converges to zero at \( x = \pm 2 \).