Fourier Series
The Fourier series expansion of a function \( f(x) \) is given by

\[
f(x) = \frac{a_0}{2} + \sum_{r=1}^{\infty} \left[ a_r \cos \left( \frac{2\pi rx}{L} \right) + b_r \sin \left( \frac{2\pi rx}{L} \right) \right]
\]

where

\[
a_r = \frac{2}{L} \int_{x_0}^{x_0+L} f(x) \cos \left( \frac{2\pi rx}{L} \right) \, dx
\]
\[
b_r = \frac{2}{L} \int_{x_0}^{x_0+L} f(x) \sin \left( \frac{2\pi rx}{L} \right) \, dx
\]

The complex Fourier series expansion is written as

\[
f(x) = \sum_{r=-\infty}^{\infty} c_r \exp \left( \frac{2\pi i rx}{L} \right),
\]

where

\[
c_r = \frac{1}{L} \int_{x_0}^{x_0+L} f(x) \exp \left( -\frac{2\pi i rx}{L} \right) \, dx
\]

Fourier Transform
Fourier transform of \( f(t) \) is defined by

\[
\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt,
\]

and its inverse by

\[
f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{i\omega t} \, d\omega.
\]

Exact equation
An exact first-degree first-order ODE is of the form

\[
A(x, y) \, dx + B(x, y) \, dy = 0
\]

and \( \frac{\partial A}{\partial y} = \frac{\partial B}{\partial x} \).

Bernoulli’s equation
Bernoulli’s equation has the form

\[
\frac{dy}{dx} + P(x)y = Q(x)y^n \text{ where } n \neq 0 \text{ or } 1
\]

This equation is solved by using the substitution \( v = y^{1-n} \).