1. Show that the Planck’s radiation law

\[ S(\lambda) = \frac{2\pi c^2 \hbar}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} \]

reduces to the classical radiation law

\[ S = \frac{2\pi c k_B T}{\lambda^4} \]

at large wavelength.

2. From the Planck’s radiation law (see above), derive the Wien equation

\[ \lambda_m T = b \]

where \( \lambda_m \) is the wavelength at which the intensity of radiation is the maximum, \( T \) is temperature and \( b \) is a constant. Find the value of \( b \), up to two significant digits. (Answer: \( 2.89775 \times 10^{-3} \) mK)

3. The energy of the valence electron of Na at near 0K is about 3 eV. What is the de Broglie wavelength of the electron? (Answer: 7.08Å)

4. The kinetic energy of a He atom is

\[ E = \frac{3}{2} k_B T, \]

find the de Broglie wavelength of the He atom at \( T = 1K \). (Answer: 1.25Å)

5. Using the Bohr-Sommerfeld quantization condition to determine

(a) the energy of an one-dimensional harmonic oscillator; (Answer: \( E_n = n\hbar\omega \))

(b) the possible radii of circular orbits of an electron in a uniform magnetic field. (Answer: \( r_n = \sqrt{n\hbar qB} \))

6. Two photons can become an electron and positron pair under certain condition. If the two photons have same energy, what is the largest wavelength of the photon in order for this to happen? (Answer: \( 2.43 \times 10^{-12} \) m)

7. Given the following stationary state wave function of a particle

\[ \psi(\vec{r}) = \frac{1}{r} e^{ikr} \]

calculate the probability current density and give the wave a physical interpretation. (Answer: \( \vec{J} = (\hbar k/mr^2)\hat{e}_r \), spherical outgoing wave.)