1. The energy and wave function of a particle confined in an infinite square potential well of the form

\[ V(x) = \begin{cases} \infty & x < 0, x > L \\ 0 & 0 < x < L \end{cases} \]

are

\[ E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}, \quad \psi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \]

respectively. Suppose a particle in this potential has an initial normalized wave function of the form

\[ \psi(x,0) = A \left[ \sin \left( \frac{\pi x}{L} \right) \right]^3. \]

(a) Calculate \( A \). This can be done without doing the integral \( \int d\theta \sin^6 \theta \).
(b) What is the form of \( \psi(x,t) \)?
(c) Calculate \( \langle x \rangle \).
(d) What is the probability that an energy measurement yields \( E_1 \).

Hint: \( \sin x = \frac{e^{ix} - e^{-ix}}{2i} \) and \((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\).

2. Consider the following square potential well

\[ V(x) = \begin{cases} \infty, & x < 0 \\ -V_0, & 0 < x < L \\ 0, & x > L \end{cases} \]

(a) For \( E < 0 \), find the equation that determines the allowed energy of the particle.
(b) What is the condition for there existing at least one bound state.
(c) Sketch the ground state wave function.
(d) What can you say about the wave function in comparison with that of a finite square potential well?

(FYP)