1. (Question 7.1, page 240 of text book) Evaluate the determinants

(a) \[
\begin{vmatrix}
  a & h & g \\
  h & b & f \\
  g & f & c
\end{vmatrix}
\],

(b) \[
\begin{vmatrix}
  1 & 0 & 2 & 3 \\
  0 & 1 & -2 & 1 \\
  3 & -3 & 4 & -2 \\
  -2 & 1 & -2 & 1
\end{vmatrix}
\],

(c) \[
\begin{vmatrix}
  gc & ge & a + ge & gb + ge \\
  0 & b & b & b \\
  c & e & b + e \\
  a & b & b + f & b + d
\end{vmatrix}
\]

2. (Question 7.2, page 241 of text book) Do the following sets of equations have non-zero solutions? If so, find them.

(a) \[
\begin{align*}
3x + 2y + z &= 0 \\
x - 3y + 2z &= 0 \\
2x + y + 3z &= 0
\end{align*}
\],

(b) \[
\begin{align*}
2x &= b(y + z) \\
x &= 2a(y - z) \\
x &= (6a - b)y - (6a + b)z
\end{align*}
\]

3. (Question 7.3, page 241 of text book) Using the properties of determinants, solve with a minimum of calculation the following equations for \(x\):

(a) \[
\begin{vmatrix}
  x & a & a & 1 \\
  a & x & b & 1 \\
  a & b & x & 1 \\
  a & b & c & 1
\end{vmatrix}
\] = 0;

(b) \[
\begin{vmatrix}
  x + 2 & x + 4 & x - 3 \\
  x + 3 & x & x + 5 \\
  x - 2 & x - 1 & x + 1
\end{vmatrix}
\] = 0.

4. (Question 7.5, page 241 of text book) Consider the matrices

(a) \[B = \begin{pmatrix}
  0 & -i & i \\
  i & 0 & -i \\
  -i & i & 0
\end{pmatrix}\],

(b) \[C = \frac{1}{\sqrt{8}} \begin{pmatrix}
  \sqrt{3} & -\sqrt{2} & -\sqrt{3} \\
  1 & \sqrt{6} & -1 \\
  2 & 0 & 2
\end{pmatrix} .\]

Are they (i) real, (ii) diagonal, (iii) symmetric, (iv) antisymmetric, (v) singular, (vi) orthogonal, (vii) Hermitian, (viii) anti-Hermitian, (ix) unitary, (x) normal?

5. Prove that the eigenvalues of an Hermitian matrix are real and the eigenvectors of an Hermitian corresponding to different eigenvalues are orthogonal.

6. (Question 7.9, page 242 of text book) \(A\) and \(B\) are real non-zero \(3 \times 3\) matrices and satisfy the equation

\[(AB)^T + B^{-1}A = 0\]

(a) Prove that if \(B\) is orthogonal then \(A\) is antisymmetric.

(b) Without assuming that \(B\) is orthogonal, prove that \(A\) is singular.
7. Consider the ordinary vectors in three dimensions \((a_x \hat{i} + a_y \hat{j} + a_z \hat{k})\) with complex components.

(a) Does the subset of all vectors with \(a_z = 0\) constitute a vector space? If so, what is its dimensions; if not, why not?
(b) What about the subset of all vectors whose \(z\) component is 1?
(c) How about the subset of vectors whose components are all equal?

8. Find the eigenvalues and eigenvectors of the following matrix:

\[
M = \begin{pmatrix}
2 & 0 & -2 \\
-2i & i & 2i \\
1 & 0 & -1
\end{pmatrix}.
\]

9. The 2 \(\times\) 2 matrix representing a rotation of the \(xy\)-plane is

\[
T = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}.
\]

Show that (except for certain special angles – what are they?) this matrix has no real eigenvalues. (This reflects the geometrical fact that no vector in the plane is carried into itself under such a rotation; contrast rotations in three dimensions.) This matrix does, however, have complex eigenvalues and eigenvectors. Find them. Construct a matrix \(S\) which diagonalizes \(T\). Perform the similarity transformation \((STS^{-1})\) explicitly, and show that it reduces \(T\) to diagonal form.

10. Let

\[
T = \begin{pmatrix}
1 & 1-i \\
1+i & 0
\end{pmatrix}.
\]

(a) Verify that \(T\) is Hermitian.
(b) Find its eigenvalues (note that they are real).
(c) Find and normalize the eigenvectors (note that they are orthogonal).
(d) Construct the unitary diagonalizing matrix \(S\), and check explicitly that it diagonalizes \(T\).