PC1134 Mathematical Methods in Physics I
Semester II, 2000/2001

TERM TEST II

7 April 2001

- This is an open book test.
- Time allowed: 90 minutes
- Answer all questions.
- Write your answers in the answer book provided.
- Full mark for each question is 10.
- You are advised to work on question 4 last, after you have attempted the other 4 questions.

![Diagram of an ellipse with labeled points](image)

Figure for question 5
1. The potential of an electrostatic field is given by
\[ \phi(r) = \frac{Q}{4\pi\varepsilon_0} \frac{e^{-\alpha r}}{r} \]
in spherical coordinates for all points in space. \( Q, \alpha \) and \( \varepsilon_0 \) are constants in the above equation. (i) Describe the equi-potential surfaces; (ii) find all components of the field \( \vec{E} = -\nabla \phi \), in either spherical coordinates or Cartesian coordinates.

2. The electric field produced by a charge distribution satisfies the Maxwell’s equation
\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]
where \( \rho \) is the charge density (Coulombs per unit volume). Find the charge density giving rise to the field
\[ \vec{E} = \frac{Q}{4\pi\varepsilon_0} \left( \frac{1}{r^2} + \frac{\alpha}{r} \right) e^{-\alpha r} \hat{e}_r \]
where \( \hat{e}_r \) is the unit vector along the radial direction in spherical coordinates. All other quantities have same meanings as in question 1.

3. Assume the following charge distribution in space,
\[ \rho(r) = -\frac{Q\alpha^2}{4\pi} \frac{e^{-\alpha r}}{r} , \]
where all quantities have same meanings as in question 1. Calculate the total charge.

4. For the charge distribution given in question 3, use the Gauss’s law or the divergence theorem, both given below, and the spherical symmetry to find the electric field at the distance \( r \) from the origin.

\[ \text{Gauss’s law: } \iiint \vec{E} \cdot d\vec{\sigma} = \frac{1}{\varepsilon_0} \iiint \rho d\tau \]

\[ \text{Divergence theorem: } \iiint \vec{E} \cdot d\vec{\sigma} = \iiint \nabla \cdot \vec{E} d\tau \]
In both cases, the volume integral is over a volume which is enclosed by a closed surface over which the surface integral is evaluated.

If the charge density that you obtained in question 2 is the same as the charge density given in question 3, does the electric field you just obtained (in this question) agree with that given in question 2? If not, what can you conclude?

5. The electric field of a point charge \( Q \) located at the origin is given by
\[ \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{e}_r \]
Calculate the flux \( \Phi \) of the electric field through the circle of radius \( a \), at a distance \( d \) from the charge, as shown in the figure on the cover page
\[ \Phi = \iint \vec{E} \cdot d\vec{\sigma} \]
Assume that the plane of the circle is perpendicular to the line joining the centre of the circle and the charge (the origin).

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