Band gap parameters of one-dimensional bicomponent nanostructured magnonic crystals

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We investigate theoretically the lowest-frequency spin waves in one-dimensional periodic arrays of alternating cobalt and permalloy nanostructures to analyze their recently measured first band gap parameters. Based on insights from numerical calculations, we formulate simple equations relating center frequencies and widths of the first band gaps of these magnonic crystals to their geometric and magnetic parameters. Analytical calculations based on these equations accord well with both the measured and the numerically generated size-dependences of the band gap parameters. Our approach permits the easy and convenient evaluation of the band gap parameters of bicomponent magnonic crystals comprising one-dimensional arrays of nanostructures of alternating magnetic materials. © 2011 American Institute of Physics. [doi:10.1063/1.3541886]

Magnonic crystals (MCs), artificial crystals of periodically modulated magnetic and geometric properties, have great technological potential as they provide control of spin waves (SWs) similar to what photonic crystals do for electromagnetic waves. For MCs, frequency band gaps of their propagating SWs are essential to the functioning of nanodevices based on them. However, the evaluation of band gap parameters, such as band gap center frequencies and widths for various MCs, is nontrivial and usually involves numerical methods such as the plane wave expansion method or the dynamical matrix approach. Recently, Wang et al. performed Brillouin light scattering measurements on MCs in the form of one-dimensional periodic arrays of alternating contacting permalloy and cobalt nanostructures, with respective widths of and which will be denoted by MPy/NCo. An advantage of MPy/NCo arrays over is that the exchange coupling at the interfaces increases the intercoupling between neighboring stripes and reduces energy loss of SW propagation. The recent Brillouin measurements on MPy/NCo arrays revealed that tunability of their magnonic band gaps can be achieved by varying the widths of the component stripes. In this study, based on insights gathered from numerical investigations of the SW modes in MPy/NCo arrays, we developed simple equations relating their first band gap parameters to their magnetic and geometric parameters. The equations allow the easy and convenient evaluation of the band gap parameters.

Figure 1 shows the schematic structure of a periodic MPy/NCo magnonic crystal. The 30 nm thick MC comprises permalloy stripes of width (M nm) and cobalt stripes of width (N nm), both of which are treated as being infinitely long in the z-direction. SW modes propagating along the length (x-direction) of the MC were calculated based on the same numerical method as that used in Ref. 5, which yielded good agreement with experimental results. Briefly, the method involves the solving (under the usual magneto-static boundary conditions) of the linearized Landau–Lifshitz equations to obtain the SW frequencies and dynamical magnetizations . Application of the Bloch theorem gives , where is the Bloch wave vector and is the lattice constant of the crystal. Calculations were based on the following magnetic parameters: saturation magnetization for permalloy; and for cobalt. Next, we discuss the observations from numerical results leading to the development of our model.

Figure 2 shows the calculated dispersion curves of the lowest-frequency SW modes of the 250Py/150Co, 250Py/250Co, and 250Py/500Co arrays. Also shown are the respective profiles plotted along the x-axis, since they are uniform along the thickness (y-direction) of these MCs. The frequencies of the lowest SW excitations in single isolated cobalt stripes of widths 150 nm, 250 nm, and 500 nm were also numerically calculated and found to be 17.3 GHz, 14.6 GHz, and 11.1 GHz, respectively. Figure 2 shows that these mode frequencies (red horizontal dashed lines) are the frequency thresholds of the corresponding MCs, below which SW modes are characterized by the resonant precessions in the permalloy stripes and the forced oscillations of the dynamical magnetizations in the cobalt stripes.

For 250Py/150Co, the 17.3 GHz frequency of the lowest mode in an isolated 150 nm wide cobalt stripe falls within the 16.9–17.9 GHz range of the dispersion branch corresponding to the seventh lowest-frequency mode [see Fig. 2(a)]. Profiles of the six modes with frequencies below 17.3 GHz were characterized by resonant precessions in the form of (sinusoidal) standing waves across the permalloy stripes, and forced oscillations of the dynamical magnetizations.

FIG. 1. (Color online) Schematic of the one-dimensional MPy/NCo MC.
within the cobalt stripes. In contrast, resonant (sinusoidal) waveforms exist in the cobalt region only for the seventh and eighth lowest-frequency modes. A similar SW behavior was observed for the 250Py/250Co and 250Py/500Co structures, for which the threshold frequencies are 14.6 GHz and 11.1 GHz, respectively.

Interestingly, our findings differ from those of previous studies,12 based on which the frequency thresholds for different MPy/NC0 structures should all correspond to the same lower boundary of the Damon–Eshbach (DE) band for a cobalt thin film. However, Fig. 2 clearly shows that the values of the frequency thresholds are different for different MPy/NC0 structures.

To analyze the parameters of the first band gap, we focus on the two lowest modes. With frequencies below the frequency threshold, oscillations of the magnetizations for these modes in the cobalt stripes are forced and driven by dynamic dipolar fields generated by the resonant precessions in the permalloy stripes. Hence, the cobalt stripes collectively serve as a medium which enhances dipolar coupling between neighboring permalloy stripes whose precessions are strongly correlated, i.e., phase locked. Analogous to the case of a single-component array of closely spaced permalloy stripes,7 enhanced coupling between our adjacent permalloy stripes modifies the sinusoidal dynamical magnetization profiles within the permalloy region and results in the dispersion of SW modes in MPy/NC0 magnonic arrays.

Figure 3 shows the calculated $m_x$ profiles for the two lowest-frequency modes at $K=0$ and $K=\pi/a$ across two unit cells. We focus on $m_x$, due to large ellipticity in the magnetization precessions ($|m_y|^2 \gg |m_x|^2$). For an isolated permalloy stripe, the $m_x$ profile for the lowest-frequency mode is approximately described by a cosine function symmetric about the stripe center.7 For $K=0$, the driving SW precessions in neighboring permalloy stripes are in phase. Thus, the generated dipolar fields over the cobalt region point in the same direction. Together with the interface exchange coupling, the resultant dipolar field aligns the dynamical magnetization in the cobalt region and results in a quasiconstant $m_x$ profile over this region [see Fig. 3(a)]. Boundary conditions at the cobalt-permalloy interfaces, viz., continuity of $m_x$ and $(A/M_T)(\partial m_x/\partial x)$,5 cause variations in $m_x$ in the permalloy stripes to be much more gradual than those in an isolated permalloy stripe. Hence the characteristic wave number of the cosine $m_x$ profile (see below) over the permalloy region becomes very small.

For the lowest mode at $K=\pi/a$, the magnetization precessions in adjacent permalloy stripes are out of phase and thus the generated dipolar fields over the cobalt region oppose each other. Owing to the driving dipolar fields and interface exchange couplings, $m_x$ within a cobalt stripe have the largest amplitude and are in opposite directions near the two interfaces of the stripe. The magnitude of $m_x$ decreases quasilinearly toward the center of the cobalt stripe, until it vanishes at the center where the two opposing dipolar fields cancel out each other. Boundary conditions at the interfaces then result in larger $m_x$ variations in the permalloy stripe (compared to the $K=0$ case) and hence, a larger characteristic wave number for the cosine $m_x$ profile in the permalloy region. The larger wave number of the DE-type resonant waveforms in the permalloy region accounts for the higher SW frequency at $K=\pi/a$ in the dispersion for the lowest mode in Fig. 2(b).

For the second lowest mode, Fig. 3(b) shows that their $m_x$ profiles within the permalloy stripes can be approximated...
by sine functions which are antisymmetric about the permalloy stripe centers. For this mode, dipolar fields within the cobalt region, generated by neighboring permalloy stripes, are in the same direction for $K = \pi / a$ and opposite directions for $K = 0$. Hence, the situation is reversed for the second lowest-frequency mode. Within the cobalt stripes, we see a quadratic constant $m_1$ profile for $K = \pi / a$ and a quasilinear $m_x$ profile for $K = 0$. The dispersion relations reveal that the mode frequency is higher at $K = 0$ than at $K = \pi / a$ (see Fig. 2).

Guided by the above physical insights, we now formulate a simple model for evaluating the first band gap parameters of the MCs with different stripe widths. At $K = \pi / a$, the lowest-frequency SW can be approximated by $m_1(x) = A_1 \cos(k_1 x)$ for $-w_1 / 2 \leq x \leq w_1 / 2$, and $m_1(x) = A_2 (1 - 2(x - w_1 / 2) / w_2)$ for $w_1 / 2 \leq x \leq (w_1 / 2) + w_2$. $A_1$ and $A_2$ are arbitrary constants and $k_1$ is the characteristic wave number for the resonant magnetization precession over the permalloy region for the lowest-frequency mode. $m_2$ over the cobalt region is described by a linear function as discussed above. Similarly, for the second lowest mode, $m_2(x) = A_3 \sin(k_2 x)$ for $-w_1 / 2 \leq x \leq w_1 / 2$ and $m_2 = A_4$ for $w_1 / 2 \leq x \leq (w_1 / 2) + w_2$. Applying boundary conditions at the interface $x = w_1 / 2$, we obtain

$$k_1 \tan \left( \frac{k_1 w_1}{2} \right) = \frac{A_{Co} M_{S,Co}}{A_{Py} M_{S,Py}} \left( \frac{2}{w_2} \right),$$

and

$$\cos \left( \frac{k_2 w_1}{2} \right) = 0, \text{ i.e. } k_2 = \pi / w_1,$$

For the lowest mode [see Fig. 3(a)], we expect $0 < k_1 < \pi / w_1$ where the left hand side of Eq. (1) is a monotonically increasing function. Replacing it by its Padé approximation of order $(2, 2)$, i.e., $k_1 \tan(k_1 w_1 / 2) = -6w_1 k_1^2 / (w_1^2 k_1^2 - 12)$, yields the solution

$$k_1 = \sqrt{\frac{12A_{Co} M_{S,Co}}{A_{Co} M_{S,Py} w_1^2 + 3A_{Py} M_{S,Co} w_1 w_2}}.$$

Thus, the wave numbers characterizing resonant magnetization precessions in the permalloy stripes are explicitly expressed in terms of $w_1$ and $w_2$. Frequencies of the SW modes were then estimated from the Kalinikos–Slavin dispersion relation

$$f(k) = \frac{\gamma_{Py} \mu_0}{2 \pi} \sqrt{\frac{2A_{Py} k^2}{\mu_0 M_{S,Py}^2}} \left( \frac{2A_{Py} k^2}{\mu_0 M_{S,Py}^2} + M_{S,Py} F(kL) \right),$$

where $k = k_1$ and $k_2$ for the lowest and second lowest modes respectively, and $L$ is the thickness of the MC. $F(kL) = 1 + P(kL) \left[ 1 - P(kL) \left[ \mu_0 M_{S,Py}^2 / (2A_{Py} k^2) \right] \right]$, where $P(kL) = 1 - [1 - \exp(-kL)] / kL$. The center frequency and width of the first band gap are thus given by $[f(k_2) + f(k_1)] / 2$ and $[f(k_2) - f(k_1)]$, respectively.

![Figure 4](http://apl.aip.org/apl/copyright.jsp)  
**FIG. 4.** (Color online) Size-dependence of the first band gap parameters. Band gap center frequency, and width for (a) $250\text{Py}/\text{NCo}$ array as a function of $N$, and (b) $250\text{Py}/150\text{Co}$ array as a function of $M$. Results of numerical and analytic calculations are denoted by solid and dashed lines, respectively, while dots represent experimental Brillouin data of Ref. 5.

Analytically calculated band gap parameters based on our simple model are compared with numerical calculations and Brillouin measurements in Fig. 4. In general, the analytical calculations accord well with the numerical and experimental results. In particular, our simple approach correctly predicts the monotonic increase/decrease in band gap width with increasing widths of the cobalt/permalloy stripes. It also accounts for the strong dependence of band gap center frequencies on the width of the permalloy stripes. The $m_1$ profiles based on the simple model were also calculated and displayed in Fig. 3. Profiles for $K = 0$ were calculated based on a similar reasoning discussed above. As can be seen, our analytical calculations are able to reproduce the general features of the numerically generated profiles.

Based on numerical studies of SWs in one-dimensional periodic arrays of alternating, contacting permalloy and cobalt stripes, we have formulated simple equations relating center frequencies and widths of the first band gaps of these MCs to their geometric and magnetic parameters. Our analytical approach offers a simple and convenient estimate of the band gap parameters of the MCs. Furthermore, our approach can be generalized to study such bicomponent MCs composed of any two magnetic materials.

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