PC1222 Fundamentals of Physics II
Focal Length of Thin Lenses

1 Purpose

• Determine the focal length of a converging lens by forming a real image of an object at various distances from the lens.
• Determine the equivalent focal length of two lenses placed in contact by forming a real image of an object at various distances from the lenses.
• Determine the focal length of a diverging lens from the equivalent focal length of two lenses placed in contact.

2 Equipment

• Optical bench, lens holders and a screen
• Light source with crossed-arrow to serve as an illuminated object
• Two converging lenses with the same focal length
• One diverging lens

3 Theory

In general, there are two different types of lenses: converging and diverging. Essentially, a lens that is thicker in the middle than at the edges is converging, and a lens that is thinner in the middle than at the edges is diverging. Therefore, a lens can be classified as converging or diverging merely by taking it between one’s fingers to see if the lens is thicker at its center than at its edge.

When a beam of light rays parallel to the optical axis is incident upon a converging lens, the rays are brought together at a point called the focal point of the lens (Figure 1). The distance from the center of the lens to the focal point is called the focal length of the lens, and it is a positive quantity for a converging lens.

When a parallel beam of light rays is incident upon a diverging lens, the rays diverge as they leave the lens (Figure 1). However, if the path of the outgoing rays are traced backward,
they appear to have emerged from a point called the focal point of the lens. The distance from the center of the lens to the focal point is called the focal length of the lens, and it is a negative quantity for a diverging lens.

Lenses are used to form images of objects. There are two possible kinds of images. The first type, called a real image, is one that can be focused on a screen. For a real image, light actually passes through the points at which the image is formed. The second type of image is called a virtual image; light does not actually pass through the points at which the image is formed and the image cannot be focused on a screen.

Diverging lenses can form only virtual images but converging lenses can form either real images or virtual images. If an object is farther from a converging lens than its focal length, a real image is formed. If the object is closer to a converging lens than the focal length, the image formed is a virtual one. Whenever a virtual image is formed, ultimately it will serve as the object for some other lens system to form a real image, Often the other system is the human eye and the real image is formed on the retina of the eye.

In the process of image formation, the distance from an object to the lens is called the object distance and the distance of the image from the lens is called the image distance. The relationship between the object distance \( u \), the image distance \( v \) and the focal length of the lens \( f \) is

\[
\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{1}
\]

Note that this equation is valid both for converging (positive \( f \)) and for diverging (negative \( f \)) lenses. Normally, the object distance is considered positive. In that sense, a positive value for the image distance means that the image is on the opposite side of the lens from the object and the image is real. A negative value for the image distance means that the image is on the same side of the lens as the object and the image is virtual.

If a lens is used to form an image of a very distant object, then the object distance \( u \) is very large. For that case, the term \( 1/u \) in equation (1) is negligible compared to the other terms \( 1/v \) and \( 1/f \) in that equation. For the case of a very distant object, equation (1) becomes

\[
\frac{1}{v} = \frac{1}{f} \quad \Rightarrow \quad v = f \tag{2}
\]
For this case, the image distance is equal to the focal length. This provides a quick and accurate way to determine the focal length of a converging lens. However, it is only applicable to a converging lens because the image must be focused on a screen. A diverging lens cannot form a real image and this technique will not work directly for a diverging lens.

If two lenses with focal lengths of \( f_1 \) and \( f_2 \) are placed in contact, the combination of the two acts as a single lens of effective focal length \( f_{12} \). The effective focal length of the two lenses in contact \( f_{12} \) is related to the individual focal lengths of the lenses \( f_1 \) and \( f_2 \) by

\[
\frac{1}{f_{12}} = \frac{1}{f_1} + \frac{1}{f_2}
\]

Equation (3) holds for any combination of converging and diverging lenses. If the individual lenses \( f_1 \) and \( f_2 \) are converging, then the effective focal length \( f_{12} \) will, of course, also be converging. If one of the lenses is converging and the other is diverging, then the effective focal length can be either converging or diverging depending on the values of \( f_1 \) and \( f_2 \). If the converging lens has a smaller magnitude than the diverging lens, then the effective focal length will be converging. This fact can be used to determine the focal length of an unknown diverging lens if it is used in combination with a converging lens whose focal length is short enough to produce a converging combination.

## 4 Experimental Procedure

### Part I: Focal length of a single lens

![Figure 2: Equipment setup.](image)

**P1.** Mount the converging lens and the screen on the optical bench. Record the manufactured focal length of the converging lens as \( f_A \) in Data Table 1.

**P2.** Move the screen until a sharp real image of a distant object (such as a distant big building) is formed on the screen. Record the distance between the lens and screen as \( v \) in Data Table 1.

**P3.** Place the light source and the screen on the optical bench 100 cm apart with the light source’s crossed-arrow object toward the screen. Place the converging lens between them as in Figure 2.
P4. Starting with the lens closer to the screen, slide the lens away from the screen to a position where a sharp image of the crossed-arrow object is formed on the screen. Measure the object distance and image distance. Record these measurements as \( u \) and \( v \) respectively in Data Table 1.

P5. Without moving the screen or the light source, move the lens to a second position closer to the light source where the image is in focus. Measure the object distance and image distance. Record these measurements as \( u \) and \( v \) respectively in Data Table 1.

P6. Repeat steps P4–P5 with light source-to-screen distances of 90 cm, 80 cm, 70 cm, 60 cm and 50 cm. For each light source-to-screen distance, find TWO lens positions where sharp images are formed. Record these measurements in Data Table 1.

Part II: Focal length of lenses in combination

P1. Replace the converging lens with a lens-combination which consists of an identical converging lens as previously used and another diverging lens. Record the manufactured focal length of the diverging lens as \( f_B \) in Data Table 2.

P2. Place the light source at some distance from the lens-combination. Move the screen to a position where a sharp image of the crossed-arrow object is formed on the screen. Measure the object distance and image distance. Record these measurements as \( u \) and \( v \) respectively in Data Table 2.

P3. Repeat step P2 for various object distances until EIGHT set of data are obtained. Record these measurements in Data Table 2.

Data Processing

Part I: Focal length of a single lens

D1. Each set of your data in Data Table 1 is an independent measurement for the focal length \( f_A \) of the converging lens. Calculate the values of the focal length \( f_A \) of the converging lens for each of the pairs of object and image distances.

D2. Determine your best experimental value for the focal length \( f_A \) of the converging lens with the corresponding uncertainty.

D3. Calculate the percentage discrepancy between the experimental value and the manufactured value for the focal length \( f_A \) of the converging lens.

**Hint:** The percentage discrepancy is defined as

\[
\text{Percentage discrepancy} = \left| \frac{\text{Experimental value} - \text{Manufactured value}}{\text{Manufactured value}} \right| \times 100\%
\]
**Part II: Focal length of lenses in combination**

**D4.** Perform a linear least squares fit to your data in Data Table 2, with the reciprocal of the image distance $1/v$ as the $y$-axis and reciprocal of the object distance $1/u$ as the $x$-axis. Determine the slope and intercept with the corresponding uncertainties of the least squares fit to the data.

**D5.** Plot a graph of the reciprocal of the image distance $1/v$ against the reciprocal of the object distance $1/u$. Also show on the graph the straight line that was obtained by the linear least squares fit to the data.

**D6.** Determine your best experimental value for the effective focal length $f_{AB}$ of the lens-combination using finite object distances with the corresponding uncertainty.

**Hint:** If $g(x, y) = x/y$, the uncertainty of $g$ is

$$
\Delta g = g \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}
$$

**D7.** Calculate a theoretical value expected for the effective focal length $f_{AB}$ of the lens-combination using their manufactured values.

**D8.** Calculate the percentage discrepancy between the experimental value and the theoretical value for the effective focal length $f_{AB}$ of the lens-combination.

**D9.** Determine your best experimental value for the focal length of the diverging lens $f_B$ with the corresponding uncertainty.

**Hint:** Denote the focal lengths of the converging lens and diverging lens as $f_A$ and $f_B$ respectively. The effective focal length $f_{AB}$ by placing these two lenses in contact is given by

$$
\frac{1}{f_{AB}} = \frac{1}{f_A} + \frac{1}{f_B}
$$

The focal length of the diverging lens and its corresponding uncertainty are given by

$$
\frac{1}{f_B} = \frac{1}{f_{AB}} - \frac{1}{f_A}
$$

$$
\Delta f_B = f_B^2 \sqrt{\left(\frac{\Delta f_{AB}}{f_{AB}^2}\right)^2 + \left(\frac{\Delta f_A}{f_A^2}\right)^2}
$$

**D10.** Calculate the percentage discrepancy between the experimental value and the manufactured value for the focal length $f_B$ of the diverging lens.
Questions

Q1. Explain why, for a given screen-to-object distance, there are two lens positions where a clear image forms.

Q2. Compare the agreement between the experimental and theoretical values for the effective focal length $f_{AB}$ of the lens-combination. Do these data suggest that equation (3) is valid for the effective focal length of two lenses in contact?

Q3. Two thin lenses are in contact. One of the lenses has a focal length of +10.0 cm when used alone. When the two are in combination, an object 20.0 cm from the lenses forms a real image 40.0 cm away from the lenses. What is the focal length of the second lens? Show your work.