**PC114X Physics IV**

*Measurements and Error Analysis*

**Instruction:** You are allowed to use Excel to perform the calculations. Submit your assignment (attached with necessary Excel spreadsheet printout) by 5p.m. on 1st February 2008 (Friday) to level 1000 Physics laboratory (S12-04-02).

1. In his famous experiment with electrons, J. J. Thomson measured the “charge-to-mass ratio” \( r = \frac{e}{m} \), where \( e \) is the electron’s charge and \( m \) its mass. A modern classroom version of this experiment finds the ratio \( r \) by accelerating electrons through a voltage \( \Delta V \) and then bending them in a magnetic field. The ratio \( r = \frac{e}{m} \) is given by the formula

\[
r = \frac{125}{32\mu_0 N^2} \frac{D^2 \Delta V}{d^2 I^2}
\]

In this equation, \( \mu_0 \) is the permeability constant of the vacuum (equal to \( 4\pi \times 10^{-7} \text{N/A}^2 \) exactly) and \( N \) is the number of turns in the coil that produces the magnetic field; \( D \) is the diameter of the field coils, \( \Delta V \) is the voltage that accelerates the electrons, \( d \) is the diameter of the electrons’ curved path and \( I \) is the current in the field coils. A student makes the following measurements:

- \( N = 72 \) (exactly)
- \( D = 661 \pm 2 \text{ mm} \)
- \( \Delta V = 45.0 \pm 0.2 \text{ V} \)
- \( d = 91.4 \pm 0.5 \text{ mm} \)
- \( I = 2.48 \pm 0.04 \text{ A} \)

Find the student’s best estimate value for charge-to-mass ratio of the electron with its uncertainty. State the final result with the appropriate number of significant figures.

2. A student wants to check the resistance of a resistor by measuring the voltage across it (\( \Delta V \)) and the resulting current through it (\( I \)) and then calculating the resistance as \( R = \frac{\Delta V}{I} \). He measures five values of \( \Delta V \) and the corresponding currents \( I \), as follows:

<table>
<thead>
<tr>
<th>Voltage ( \Delta V ) (V)</th>
<th>11.2</th>
<th>13.4</th>
<th>15.1</th>
<th>17.7</th>
<th>19.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current ( I ) (A)</td>
<td>4.67</td>
<td>5.46</td>
<td>6.28</td>
<td>7.22</td>
<td>7.89</td>
</tr>
</tbody>
</table>

Estimate the resistance of the resistor \( R \) and its standard deviation, and state the final result with the appropriate number of significant figures.
3. The spectrum from a hot gas of an element consists of discrete wavelengths that are characteristic of the element. In 1885 in an attempt to understand these spectra, Johann Balmer published an empirical relationship that described the visible spectrum of hydrogen:

\[
\frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \quad n = 3, 4, 5, 6, \ldots
\]

where \( R \) is a constant called the Rydberg constant, \( \lambda \) is the wavelength and \( n \) is an integer that takes the value greater than 2.

In an experiment to determine the Rydberg constant using the first four visible lines from the hydrogen spectrum, the following data were obtained:

<table>
<thead>
<tr>
<th>Colours</th>
<th>Violet</th>
<th>Blue</th>
<th>Blue-Green</th>
<th>Red</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda (\times 10^{-7} \text{ m}) )</td>
<td>4.114</td>
<td>4.337</td>
<td>4.863</td>
<td>6.561</td>
</tr>
<tr>
<td>( n )</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Estimate the value of the Rydberg constant and state the final result with the appropriate number of significant figures.

4. In a decay process of radioactive material, the variation of the number of particles with time is given by equation

\[
N = N_0 e^{-\lambda t}
\]

where \( N_0 \) is the number of particles present at initial time \( t = 0 \), \( N \) is the number of particles remaining after a time \( t \) and \( \lambda \) is a constant.

In an experiment to determine \( \lambda \) and \( N_0 \) for a certain radioactive material, the following values for \( t \) and \( N \) were obtained.

<table>
<thead>
<tr>
<th>( t ) (s)</th>
<th>4000</th>
<th>8000</th>
<th>12000</th>
<th>16000</th>
<th>20000</th>
<th>24000</th>
<th>28000</th>
<th>32000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>2310</td>
<td>1250</td>
<td>681</td>
<td>382</td>
<td>198</td>
<td>115</td>
<td>60</td>
<td>35</td>
</tr>
</tbody>
</table>

(a) Linearize the above equation by showing what are the independent variable \((x)\), dependent variable \((y)\), slope \((m)\) and the intercept \((c)\);

(b) Make a least-squares fit for the given data to your linearized equation in (a) and find the best estimates for \( N_0 \) and \( \lambda \) with their corresponding uncertainties. State the final results with the appropriate number of significant figures.

(c) Plot a suitable graph based on your linearized equation above. Also show on the graph the straight line that was obtained by fitting the data via the linear least-squares fit.
5. Consider an experiment in determining the electromotive force and internal resistance of a dry battery where the resistance of the resistance box is varied and the current flowing in the circuit is measured. The relationship between the resistance $R$ and the current $I$ is given by

$$\varepsilon = I(R + r)$$

where $\varepsilon$ and $r$ are the electromotive force and internal resistance of the dry battery. The measurements of $R$ and $I$ are as follow:

<table>
<thead>
<tr>
<th>$R$ (Ω)</th>
<th>500</th>
<th>480</th>
<th>460</th>
<th>440</th>
<th>420</th>
<th>400</th>
<th>380</th>
<th>360</th>
<th>340</th>
<th>320</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ (A)</td>
<td>0.0145</td>
<td>0.0154</td>
<td>0.0160</td>
<td>0.0165</td>
<td>0.0173</td>
<td>0.0178</td>
<td>0.0190</td>
<td>0.0195</td>
<td>0.0205</td>
<td>0.0220</td>
</tr>
</tbody>
</table>

Assuming that the uncertainty in the resistance measurement is negligible.

(a) Linearize the above equation by showing what are the independent variable ($x$), dependent variable ($y$), slope ($m$) and the intercept ($c$);

(b) Make a least-squares fit for the given data to your linearized equation in (a) and find the best estimates for $\varepsilon$ and $r$ with their corresponding uncertainties. State the final results with the appropriate number of significant figures.

(c) Plot a suitable graph based on your linearized equation above. Also show on the graph the straight line that was obtained by fitting the data via the linear least-squares fit.