1 Purpose

- Investigate the phase relationships among the voltages across the resistor, the capacitor and the inductor in circuits.
- Determine the value of the value of $L$ and $r$ for an unknown real inductor in an LR circuit.
- Determine the value of the capacitance of a capacitor in an RC circuit.

2 Equipment

- Sine-wave signal generator
- Resistance box consisting of resistors 100 $\Omega$, 220 $\Omega$ and 220 $\Omega$ connected in series
- An unknown real inductor
- A capacitor with capacitance of 1.00 $\mu$F
- Digital multi-meter

3 Theory

LR Circuit

Consider two circuits shown in Figure 1 in which a sine wave generator of frequency of $f$ is connected separately to resistor $R$ and then to a pure inductance $L$. The generator is assumed to have a maximum voltage of $V$ across the resistor in circuit (a). It will also produce a maximum voltage of $V$ across the inductor in circuit (b). The voltage across the resistor is related to the current by a relationship like that for direct current circuits, which is

$$V_R = IR \quad (1)$$

If $L$ is the inductance (units H) and $\omega = 2\pi f$ is the angular frequency of the generator in rad/s, then the following relationship exists between the voltage $V_L$ and the current $I$

$$V_L = I\omega L \quad (2)$$
The quantity $\omega L$ is called the inductive inductance and it has units of $\Omega$.

Figure 1: Generator and resistor and generator and inductor. Phase relationship between the voltage and current and phasor diagrams of the phase relationships.

When an alternating current or voltage is measured in the laboratory on a meter, the number read for the current or voltage must be a time-averaged value. Meters are normally calibrated so that they respond to the root-mean-square value of the current or voltage. A root-mean-square of voltage is designated as $V_{\text{rms}}$. The relationship between $V_{\text{rms}}$ and $V$ the maximum voltage is $V_{\text{rms}} = 0.707V$. All measurements in this laboratory will be rms values.

Also shown in Figure 1 below each circuit is a graph of the current and voltage across the element for one full period. The graph for the case of the resistor indicates that the resistor current $I_R$ and the resistor voltage $V_R$ are in phase. For the inductor, the graph shows that the inductor current $I_L$ and the inductor voltage $V_L$ are $90^\circ$ out of phase, with the voltage leading the current by $90^\circ$.

Shown at the bottom of Figure 1 is a diagram called a phasor diagram. Its purpose is also to show the phase relationship. The phasors are vectors drawn with length proportional to the value of the represented quantity and they are assumed to be rotating counterclockwise with the frequency of the generator. At any time, a projection of one of the rotating vectors on the $y$ axis is the instantaneous value of that quantity. Because the resistor current and voltage
are in phase, the phasors are in the same direction. For the inductor, the vector representing the inductor voltage is $90^\circ$ ahead of the vector representing the current.

![Image](image_url)

Figure 2: Series circuit of resistor and inductor and associated phasor diagram.

Consider now the circuit obtained by placing a pure inductance $L$ having no resistance and a resistor $R$ in series with a sine wave generator of voltage $V$ shown in Figure 2. For this circuit, the current $I$ is the same at every instant of time in all three circuit elements. Also given in Figure 2 is a phasor diagram in which only the voltages are shown. The phasor representing current (which is not shown) would be in the direction of the phasor labeled $V_R$ because the current and the resistor voltage are in phase. Note that the inductor voltage $V_L$ is $90^\circ$ ahead of the resistor voltage $V_R$ and the generator voltage is angle $\phi$ ahead of $V_R$. The phasor diagram shows that the generator voltage $V$ is the vector sum of $V_R$ and $V_L$. In equation form, the phasor diagram states

$$V = \sqrt{V_L^2 + V_R^2}$$  \hspace{1cm} (3)

The phasor diagram also shows that the phase angle $\phi$ is related to the voltages $V_L$ and $V_R$, and thus to the resistances $R$ and $\omega L$ through equations (1) and (2). The relationship is given by

$$\tan \phi = \frac{V_L}{V_R} = \frac{\omega L}{R}$$  \hspace{1cm} (4)

Note that equation (4) is strictly valid only for a pure inductor that has no resistance. Real inductors have both an inductance $L$ and an internal resistance $r$, and can be represented by a pure inductance $L$ in series with a pure resistance $r$. In Figure 3, a real inductor is shown in series with a resistor $R$ and a generator of voltage $V$. The voltage between points $A$ and $B$ is the generator voltage $V$ and the voltage between $A$ and $C$ is the resistor voltage $V_R$. Between the points $B$ and $C$ is the combined voltage across the inductance $L$ and the internal resistance $r$. This voltage will be referred to as $V_{\text{ind}}$. There is some voltage $V_L$ across $L$ and some voltage $V_r$ across $r$. However, there can be no direct measurement of $V_L$ and $V_r$. The only quantity that can be measured is $V_{\text{ind}}$ which is the vector sum of $V_L$ and $V_r$. A phasor diagram for the circuit is also shown in Figure 3.

Applying the law of cosine to the triangle formed by $V$, $V_R$ and $V_{\text{ind}}$ leads to

$$\cos \phi = \frac{V^2 + V_R^2 - V_{\text{ind}}^2}{2VV_R}$$  \hspace{1cm} (5)
Figure 3: Series circuit of inductance $L$ and internal resistance $r$, a resistor $R$ and a generator of voltage $V$. Also shown is the phasor diagram of the voltages.

The phasor diagram in Figure 3 shows that voltages $V_L$ and $V_r$ can be determined from $V$, $V_R$ and $\phi$ by

$$V_L = V \sin \phi \quad \text{and} \quad V_r = V \cos \phi - V_R$$ \hspace{1cm} (6)

The current $I$ is the same in all the elements of the circuit and it can be related to the voltage across each element by the following equations:

$$V_L = I\omega L, \quad V_R = IR, \quad V_r = Ir$$ \hspace{1cm} (7)

With $\phi$, $V_L$ and $V$, determined from equations (5) and (6), equation (7) can be used to solve for $\omega L$ and $r$ by eliminating $I$ to get

$$\omega L = R \frac{V_L}{V_R} \quad r = R \frac{V_r}{V_R}$$ \hspace{1cm} (8)

**RC Circuit**

Figure 4: RC circuit and the phasor diagram for the voltage across each element.

A series circuit consisting of a capacitor $C$, a resistor $R$ and a sine wave generator of frequency $f$ is shown in Figure 4. Also shown in the figure is a phasor diagram for the
generator voltage $V$, the voltage across the resistor $V_R$ and the capacitor voltage $V_C$. It is assumed that all voltages discussed are root-mean-square values. The voltages $V_R$ and $V_C$ are $90^\circ$ out of phase, and the voltages $V$, $V_R$ and $V_C$ form a triangle as shown in Figure 4. Therefore, the equation relating the magnitudes of the measured voltages in an $RC$ circuit is

$$V = \sqrt{V_C^2 + V_R^2}$$ (9)

Note that equation (9) is valid for only a pure capacitor with no resistive component. If measurements on a real capacitor shown agreement with equation (9), it would indicate that the capacitor has no significant resistive component.

In an $RC$ circuit, the current $I$ is the same in each element of the circuit. The relationships between the voltage and the current for the resistor and capacitor are

$$V_R = IR \quad \text{and} \quad V_C = \frac{I}{\omega C}$$ (10)

The quantity $1/\omega C$ is called the capacitive reactance and it has units of ohms. If the current is eliminated between the two equations in (10), an equation for $C$ is given by

$$C = \frac{1}{\omega R} \frac{V_R}{V_C}$$ (11)

Thus a value for the capacitance of an unknown capacitor can be determined from equation (11) if $\omega$ and $R$ are known and $V_R$ and $V_C$ are measured.

**LRC Circuit**

![LRC Circuit Diagram](image)

Figure 5: LRC circuit with voltmeter in the four positions to measure voltage across each element of the circuit. Also shown is a phasor diagram of all the relevant voltages.
Consider a series LRC circuit shown in Figure 5 with a sine wave generator of voltage $V$, a resistor $R$, a capacitor $C$ and an inductor have inductance $L$ and resistance $r$. Note that the capacitor is assumed to have no resistance. Also shown in Figure 5 is the phasor diagram for the voltages $V$, $V_R$, $V_C$, $V_L$ and $V_r$. The figure shows that $V_L$ and $V_C$ are 180° out of phase, and $V_R$ and $V_r$ are in phase. The quantities $V_L - V_C$, $V$ and $V_R + V_r$ form a triangle and the relationship between them is given by

$$V = \sqrt{(V_L - V_C)^2 + (V_R + V_r)^2} \quad (12)$$

It is impossible to measure either $V_L$ and $V_r$ directly. The only voltage associated with the inductor that can be measured experimentally is shown in Figure 5 as $V_{ind}$ and it is the vector sum of the voltages $V_L$ and $V_r$. The relationships between $V_{ind}$, $V_L$ and $V_r$ is shown in Figure 3. The figure shows that the voltages $V_{ind}$, $V_L$ and $V_r$ obey the relationship

$$V_{ind}^2 = V_L^2 + V_r^2 \quad (13)$$

The current $I$ is the same in each element of the circuit, and $V_L$ and $V_r$ can be expressed as

$$V_L = I (\omega L) \quad \text{and} \quad V_r = Ir \quad (14)$$

If equations (13) and (14) are combined, and the current $I$ is eliminated, it can be shown that

$$V_L = V_{ind} \frac{\omega L}{\sqrt{(\omega L)^2 + r^2}} \quad \text{and} \quad V_r = V_{ind} \frac{r}{\sqrt{(\omega L)^2 + r^2}} \quad (15)$$

Assuming that $\omega$, $L$ and $r$ are known, equation (15) can be used to determine $V_L$ and $V_r$ if $V_{ind}$ is measured. These values of $V_L$ and $V_r$ combined with measured values of $V_R$ and $V_C$ can be used in equation (12) to verify the relationship between these quantities and the measured sine wave generator voltage $V$.

## 4 Experimental Procedure

### Part I: LR Circuit

**P1.** Connect the inductor in series with the sine-wave signal generator and a resistance box to form a circuit like that of Figure 2. Set the generator voltage at 6 V and a frequency of 800 Hz. Set the resistance box to a value of 100 Ω and record that value as $R$ and the frequency $f$ in the Data Table 1.

**P2.** Using digital multi-meter, measure the generator voltage $V$, the inductor voltage $V_{ind}$ and the resistor voltage $V_R$. Record these values in the Data Table 1.

**P3.** Repeat the above steps for $R$ of 220 Ω, 320 Ω, 440 Ω and 540 Ω. Even though the generator voltage is left at 6 V, the generator output might change slightly in response to the changes in $R$. Therefore, be sure to measure all three voltages for each value of $R$. 
**Part II: RC Circuit**

**P1.** Connect the capacitor provided in series with the sine-wave signal generator and a resistance box to form a circuit like that of Figure 4. Set the generator voltage at 6 V and a frequency of 200 Hz. Set the resistance box to a value of 100 Ω and record that value as $R$ and the frequency $f$ in the Data Table 2.

**P2.** Using digital multi-meter, measure the generator voltage $V$, the capacitor voltage $V_C$ and the resistor voltage $V_R$. Record these values in the Data Table 2.

**P3.** Repeat the above steps for $R$ of 220 Ω, 320 Ω, 440 Ω and 540 Ω. Even though the generator voltage is left at 6 V, the generator output might change slightly in response to the changes in $R$. Therefore, be sure to measure all three voltages for each value of $R$.

**P4.** Record the manufactured value for the capacitance of your capacitor as $C_k$ in Data Table 2.

**Part III: LRC Circuit**

**P1.** Construct a series $LRC$ circuit like the one shown in Figure 5 using the given capacitor and inductor, a resistance box and the sine-wave signal generator.

**P2.** Set the generator voltage at 6 V and a frequency of 200 Hz. Set the resistance box to a value of 220 Ω and record that value as $R$ and the frequency $f$ in the Data Table 3.

**P3.** Using digital multi-meter, measure the generator voltage $V$, the inductor voltage $V_{ind}$, the capacitor voltage $V_C$ and the resistor voltage $V_R$. Record these values in the Data Table 3.

**P4.** Repeat step **P3** with the resistance box set to $R = 440 \, \Omega$. Even though the generator voltage is left at 6 V, the generator output might change slightly in response to the changes in $R$. Therefore, be sure to measure all four voltages for each value of $R$.

**P5.** Repeat the above steps **P1–P4** with the generator frequency set to 600 Hz and 800 Hz respectively.
5 Data Processing

Part I: LR Circuit

D1. For each set of your data in Data Table 1, calculate the values of inductance $L$ and internal resistance $r$.

D2. Determine your best experimental values for the inductance $L$ and internal resistance $r$ with the corresponding uncertainties.

Part II: RC Circuit

D1. For each set of your data in Data Table 2, calculate the value of capacitance $C$.

D2. Determine your best experimental value for the capacitance $C$ with the corresponding uncertainty.

D3. Use percentage discrepancy to compare your experimental value for the capacitance $C$ with the manufactured value.

**Hint:** The percentage discrepancy is defined as

$$\text{Percentage discrepancy} = \frac{|\text{Experimental value} - \text{Manufactured value}|}{\text{Manufactured value}} \times 100\%$$

D4. Calculate value of the quantity $\sqrt{V_C^2 + V_R^2}$ for each case in Data Table 2. Calculate the percentage discrepancy in each of these values compared to the measured values of the generator voltage $V$.

Part III: LRC Circuit

D1. For each set of your data in Data Table 3, calculate the values of $V_L$ and $V_r$.

D2. Calculate the value of the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$ for each set of your data in Data Table 3.

D3. Use percentage discrepancy to compare each of your values of the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$ to the corresponding measured value of the generator voltage.
Questions

Q1. Comment on the precision of your measurement of $L$ and $r$. State the evidence for your comment.

Q2. Construct scaled phasor diagram for each of the five cases in Data Table 1. Using a protractor, measure the angle $\phi$ of the constructed triangle of $V$, $V_R$ and $V_{\text{ind}}$. Compare it with the calculated value of $\phi$ for each of the phasor diagrams. Calculate the percentage error in the value of $\phi$ from the diagram compared to the calculated value.

Q3. Consider the LR circuit that you measured with $R = 320 \Omega$. Calculate the current through each of the element ($R$, $L$ and $r$) and comment on their agreement.

Q4. Comment on the agreement between the measured generator voltage $V$ and the quantity $\sqrt{V_C^2 + V + R^2}$ for the RC circuit data.

Q5. Do your results for question Q4 confirm that the capacitor has no resistance? State specifically how the data either do or do not confirm this expectation.

Q6. State carefully your evaluation of the precision of your measurements of the value of the capacitor in the RC circuit. State the evidence for your opinion.

Q7. Comment on the accuracy of your measurements of the capacitance in the RC circuit.

Q8. Comment on the agreement between the measured generator voltage $V$ and the quantity $\sqrt{(V_L - V_C)^2 + (V_R + V_r)^2}$ in the LRC circuit.

Q9. Do your results for question Q8 confirm the phasor diagram of Figure 5 as a correct model for the addition of the voltages in an LRC circuit? State why they do or do not confirm the model.