1. Since \( 0 = [A, BB^{-1}] = B[A, B^{-1}] + [A, B]B^{-1} \) we have \( [A, B^{-1}] = -B^{-1}[A, B]B^{-1} \).

2. The oscillator will remain in the instantaneous ground state. In view of

\[ H = \hbar \omega \left( A^+ - \frac{\Omega^*}{\omega} \right) \left( A - \frac{\Omega}{\omega} \right) - \frac{1}{2} \frac{|\Omega|^2}{\omega} \]

the instantaneous ground state is the ground state of the oscillator with \( \langle A \rangle = \frac{\Omega}{\omega} \), \( \langle A^+ \rangle = \frac{\Omega^*}{\omega} \), so that

\[ \rho = e^{-\frac{1}{2}(A^+-\frac{\Omega^*}{\omega})(A-\frac{\Omega}{\omega})} \]

with \( \frac{\partial}{\partial t} = \frac{\partial}{\partial \Omega} \) applies at intermediate times and

\[ p(T) = e^{-\frac{1}{2}(A^+-\frac{\Omega^*}{\omega})(A-\frac{\Omega}{\omega})} \]

is the statistical operator (= projector to the ground state) at time \( T \).

(a) With \( \langle 0 | A^+ = 0 \rangle \) and \( \langle 0 | A = 0 \rangle \), we have

\[ \langle 0 | p(T) | 0 \rangle = e^{-\frac{1}{2}|\Omega|^2/\omega^2} \]

for the probability of finding the oscillator in the ground state \( |0\rangle \)

\( \Omega \to 0 \), when looking for \( |1\rangle \)
at time \( T \).
(b) We have \(|11\rangle = A^+ 10\rangle\), \(<11\rangle = <01A, 
so that \(\text{prob}(\text{in }11\rangle \text{ at } T) = <01A \rho(T) A^+ 10\rangle\)
where \(A \rho(T) A^+ = AA^+ \rho(T) + A[A \rho(T), A^+]\)
\[= AA^+ \rho(T) + A \frac{\partial}{\partial A} \rho(T)\]
\[= AA^+ \rho(T) - A(A^+ - \frac{\omega^2}{\omega}) \rho(T)\]
\[= (\frac{\omega^2}{\omega}) A \rho(T)\]
\[= (\frac{\omega^2}{\omega}) (\rho(T) A + [A, \rho(T)]\]
\[= (\frac{\omega^2}{\omega}) (\rho(T) A + \frac{\partial}{\partial A} \rho(T)\]
\[= (\frac{\omega^2}{\omega}) (\rho(T) A - \rho(T) (A - \frac{\omega^2}{\omega}))\]
\[= \frac{\omega^2}{\omega} \rho(T),\]
so that \(<01A \rho(T) A^+ 10\rangle\)
\[= |\frac{\omega^2}{\omega}|^2 <01\rho(T) 10\rangle\]
\[= |\frac{\omega^2}{\omega}|^2 e^{-15\omega^2/\omega^2} .\]

Note: There are quite a few different methods for getting this result, such as making use of the explicit Gaussian wave functions.
3. We have \( V_2(\mathbb{R}) = \mathcal{U} \cdot \frac{\partial}{\partial t} V_1(\mathbb{R}) e^{i\alpha \cdot \mathbf{P}/\hbar t} \)

\[ = W^+ V_1(\mathbb{R}) W \]

with the unitary \( W \) commuting with \( H_0 = \frac{\hbar}{2m} \mathbf{p}^2 \) and therefore also with \( \mathcal{G} = (E - H_0 + i\varepsilon)^{-1} \). In the equation for the transition operator to \( V_2 \),

\[ T_2 = V_2 + V_2 \mathcal{G} T_2, \]

we have then

\[ T_2 = W^+ V_1 W + W^+ V_1 \mathcal{G} W T_2 \]

or

\[ W T_2 W^+ = V_1 + V_1 \mathcal{G} W T_2 W^+ \]

so that \( W T_2 W^+ \) obeys the equation for the transition operator to \( V_1 \):

\[ T_1 = V_1 + V_1 \mathcal{G} T_1. \]

It follows that \( W T_2 W^+ = T_1 \), or

\[ T_2 = W^+ T_1 W. \]
Question 4.6

Write answers on this side of the paper only.

14. (a) In \( T = V + V \mathcal{G} V + V \mathcal{G} V + \cdots \), every term has one \( V \) to the left and one \( V \) to the right, so that \( T1 \) is a linear combination of \( |s1> \) and \( |s2> \), and \( <1T \) is a superposition of \( <s1,1 \) and \( <s2,1 \). Therefore,

\[
T = (|s1>, |s2>) \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} (\begin{pmatrix} <s1,1 \\ <s2,1 \end{pmatrix})
\]

(b) Taking matrix elements of \( T = V + V \mathcal{G} T \), we get

\[
T_{11} = <s1,1 | T | s1,1> = V_1 + V_1 <s1,1 | \mathcal{G} | s1,1>
\]

\[
= V_1 + V_1 \mathcal{G} (|s1> T_{11} + |s2> T_{21})
\]

\[
= V_1 + V_1 (G_{11} T_{11} + G_{12} T_{21})
\]

with \( G_{11} = <s1,1 | G | s1,1> \), \( G_{12} = <s1,1 | G | s2,1> \). Likewise

\[
T_{21} = V_2 G_{21} T_{11} + V_2 G_{22} T_{21}
\]

\[
T_{12} = V_1 G_{11} T_{12} + V_1 G_{12} T_{22}
\]

\[
T_{22} = V_2 + V_2 G_{21} T_{12} + V_2 G_{22} T_{22}
\]

We solve for \( T_{11}, T_{12}, T_{21}, \) and \( T_{22} \) and get
Question 5/6

Write answers on this side of the paper only.

\[
\begin{pmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{pmatrix} = \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix} \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \right]^{-1} \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}
\]

\[
= \frac{1}{\text{DET}} \begin{pmatrix} V_1 - V_1 V_2 G_{22} & V_1 G_{12} - V_2 \\ V_2 G_{21} & V_1 - V_2 V_1 G_{11} \end{pmatrix}
\]

with \( \text{DET} = (1 - V_1 G_{11})(1 - V_2 G_{22}) - V_1 G_{12} V_2 G_{21} \).

---

5. When company \( j = 2 \) and \( j' = 1 \) from \( j = \frac{3}{2} \) and \( j = \frac{1}{2} \), we have \( |j = m = 2 > = |\frac{3}{2}; \frac{3}{2} > \) and \( \text{short for } j = \frac{3}{2}, m = \frac{3}{2} \).

   \text{short for } j = \frac{1}{2}, m = \frac{1}{2} \)

so that \( |j = 2, m = 1 > \propto \begin{pmatrix} J_{+} & J_{-} \end{pmatrix} \begin{pmatrix} 1 \frac{3}{2}; \frac{3}{2} > \\
1 \frac{3}{2}; -\frac{1}{2} > \end{pmatrix} \)

\[= |\frac{1}{2}; \frac{1}{2} > \propto h \sqrt{\left(\frac{3}{2} - \frac{3}{2}\right)\left(\frac{3}{2} - \frac{3}{2}\right)}
\]

\[+ |\frac{1}{2}; -\frac{1}{2} > \propto h \sqrt{\left(\frac{1}{2} - \frac{1}{2}\right)\left(\frac{1}{2} - \frac{1}{2}\right)}
\]

\[= |\frac{1}{2}; \frac{1}{2} > h \sqrt{3} + |\frac{3}{2}; -\frac{1}{2} > h.
\]

The state with \( j = 1, m = 1 \) is orthogonal to this, but a superposition of the same two basis. Properly normalized, we have

\[|j = 1, m = 1 > = \left(\frac{\sqrt{3}}{2}; \frac{1}{2} > \sqrt{3} - \frac{1}{2} \frac{1}{2} > \right)/2.
\]

Another application of \( J_{-} \) gives...
Write answers on this side of the paper only.

\[
\begin{align*}
  J_{-1} |j=1, m=1> &= \sqrt{(1+1)(1-1+1)} \\
  &= \left( J_{-1} |\frac{3}{2}; -\frac{1}{2} > + \sqrt{3} - J_{-1} |\frac{1}{2}; \frac{1}{2} > \right) / 2 \\
  &= \left( |\frac{1}{2}; -\frac{1}{2} > + \sqrt{3} - 1 - |\frac{1}{2}; \frac{1}{2} > \right) / 2 \\
  &= \left( |\frac{1}{2}; -\frac{1}{2} > - |\frac{1}{2}; \frac{1}{2} > \right) \hbar, \\
  \text{So that } |j=1, m=1> &= \left( |\frac{1}{2}; -\frac{1}{2} > - |\frac{1}{2}; \frac{1}{2} > \right) / \sqrt{2} \\
  \end{align*}
\]

(a) These probabilities are

0 for \( m_2 = \frac{3}{2} \) and \( m_2 = -\frac{3}{2} \),

\( \frac{1}{2} \) for \( m_2 = \frac{1}{2} \) and \( m_2 = -\frac{1}{2} \).

(b) These probabilities are \( \frac{1}{2} \) each.

(c) This probability is \( \frac{1}{2} \).