We get successively,

\[ AD = ABC = C, \ AE = AAC = BC = D; \]
\[ BA = BAA = Ball = 1, \ BB = BAA = A; \]
\[ BD = BBC = AC = E, \ BE = BAC = C; \]
\[ CD = CCA = A, \ CE = CCB = B; \]
\[ DA = CAA = CB = E, \ DB = CAB = C, \ DC = BCC = B; \]
\[ DD = CABC = 1, \ DE = BCC = BB = A; \]
\[ EA = CBA = C, \ EB = CBB = CA = D, \ EC = ACC = A, \]
\[ ED = ACCA = AA = B, \ EE = CBAC = 1; \]

so that the completed table is

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>B</td>
<td>1</td>
<td>E</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>1</td>
<td>A</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>E</td>
<td>C</td>
<td>B</td>
<td>1</td>
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<tr>
<td>E</td>
<td>E</td>
<td>C</td>
<td>D</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

Subgroup with three elements: \( \{1, A, B\} \); subgroup with two elements: \( \{1, C\}; \{1, D\}; \{1, E\} \).

...all together

2) (1) Clearly, for any \( g \), and \( g' \), the product \( gg' \) is also a triplet of three real numbers.

(2) There is a unique neutral element \((0,0,0)\).

(3) There is a unique inverse to \( g=(a,b,c) \), namely \( g^{-1}=(-a,-b,-c+ab) \) such that \( gg^{-1}=g^{-1}g=(0,0,0) \).
(4) Composition is associative:

\[(g_1 * g_2)g_3 = (a_1 + a_2 + a_3, b_1 + b_2 + b_3, c_1 + c_2 + a_1b_2 + (a_2b_3) + (a_3b_2))\]

\[= (a_1 + a_2 + a_3, b_1 + b_2 + b_3, c_1 + c_2 + a_2b_3 + a_3b_2 + a_1b_2)\]

\[= g_1 (g_2 * g_3).

In view of properties (1)-(4), \( G \) is a group, indeed.
It is not Abelian because we have

\[(1, 1, 1) (0, 1, 0) = (1, 1, 1) \neq (1, 1, 0) = (0, 1, 0)(1, 0, 0), \]
for example.

\[
\text{[5] For } F(s) = \int_0^\infty \frac{e^{-at} \cos(\omega t) - \cos(\omega t)}{t} dt
\]

the integral in question is obtained for \( F(s=0), \)
and we have

\[
\frac{d}{ds} F(s) = \int_0^\infty dt \ e^{-st} [\cos(\omega t) - \cos(\omega t)]
\]

\[= \frac{\Delta}{s^2 + \omega^2} - \frac{s^2}{s^2 + \omega^2} = \frac{\Delta}{s^2 + \omega^2} \ln \frac{s^2 + \omega^2}{s^2 + \omega^2}.
\]

In conjunction with \( F(s \to \infty) = 0, \) this implies

\[
F(0) = \ln \frac{\sqrt{\Delta^2 + \omega^2}}{\sqrt{s^2 + \omega^2}}
\]

no real

\[
F(0) = \int_0^\infty \frac{\cos(at) - \cos(bt)}{t} dt = \ln \frac{b}{a}.
\]
Question 2.3

Write answers on this side of the paper only.

4] For \( f(t) = \int_0^t J_0(t-t') J(t') \) we have

\[
F(s) = \left( \frac{1}{\sqrt{1+s^2}} \right)^2 \quad \text{because of the convolution.}
\]

But \( \frac{1}{\sqrt{1+s^2}} \) is the Laplace transform of \( \sin t \),
so that \( f(t) = \sin t \) follows,

\[
\int_0^t dt' J_0(t-t') J_0(t') = \sin t.
\]

5] Making use of the periodicity, we get

\[
F(s) = \int dt e^{-st} f(t) = \int dt e^{-st} f(t+T)
\]

\[
= \int_T^\infty dt e^{-s(t-T)} f(t) = e^{sT} \left[ F(s) - \int_0^T dt e^{-st} f(t) \right],
\]

which we solve for \( F(s) \) to arrive at

\[
F(s) = \frac{1}{1-e^{-sT}} \int_0^T dt e^{-st} f(t), \quad \text{indeed.}
\]

Likewise we first establish

\[
G(s) = e^{sT} \left[ \int_0^T dt e^{-st} g(t) \right] - G(s)
\]

and then

\[
G(s) = \frac{1}{1+e^{-sT}} \int_0^T dt e^{-st} g(t).
\]