Problem 1 (15 marks)
Function \( z(x, y) \) obeys the quasi-linear partial differential equation (qLPE)
\[
\left( x \frac{\partial}{\partial x} - y \frac{\partial}{\partial y} \right) z(x, y) = 2z(x, y).
\]
Determine the general solution of this qLPE and the special solution for \( z(x, x) = 1 \).

Problem 2 (15 marks)
The general solution of a certain qLPE for \( z(x, y) \) is of the form
\[
z(x, y) = \tan \left( x + u(x^2 + y) \right) \text{ with arbitrary } u(\cdot).
\]
State this qLPE.

Problem 3 (20 marks)
For the Hamilton function
\[
H = \frac{p^2}{2m} - \frac{m}{2} \gamma^2 x^2 \quad \text{with constant } m > 0 \text{ and } \gamma > 0
\]
find the phase-space density \( \rho(t, x, p) \) in terms of \( \rho_0(x, p) = \rho(t = 0, x, p) \).

Problem 4 (25 marks)
What is the smallest value that you can get for
\[
\int_0^1 dx \left[ \frac{d}{dx} y(x) \right]^2
\]
if the permissible \( y(x) \) are restricted by
\[
y(0) = y(1) = 0 \quad \text{and} \quad \int_0^1 dx y(x) = 2?
\]

Problem 5 (25 marks)
A point-like object \([\text{mass } m, \text{ position vector } \mathbf{r} \equiv (x, y, z)]\) is moving without friction on the paraboloid specified by \( 2z = \kappa(x^2 + y^2) \) with \( \kappa > 0 \), while the gravitational force \( m\mathbf{g} \equiv (0, 0, -mg) \) is acting. Use polar coordinates in the \( x, y \)-plane, that is \( (x, y) = (s \cos \varphi, s \sin \varphi) \), and find the Lagrange function \( L(s, \varphi, \dot{s}, \dot{\varphi}) \). Then determine the corresponding Hamilton function.