Problem 1 (20 marks)
A harmonic oscillator is in the coherent state described by the ket $|a\rangle$. Determine the expectation values of position $X$ and momentum $P$ and their spreads $\delta X$ and $\delta P$. How large is their product $\delta X \delta P$?

Problem 2 (20 marks)
Orbital angular momentum: If the system is in an eigenstate of $\vec{L}^2$ with eigenvalue $2\hbar^2$, what are the possible outcomes when a measurement of $L_1L_2 + L_2L_1$ is performed?

Problem 3 (30 marks)
A harmonic oscillator (natural frequency $\omega$, ladder operators $A$ and $A^\dagger$) is perturbed by a potential proportional to $i(A^\dagger^2 - A^2)$, so that the Hamilton operator is

$$H = \hbar \omega A^\dagger A + i\hbar \Omega(A^\dagger^2 - A^2) \quad \text{with } |\Omega| < \frac{1}{2}\omega.$$  

Introduce new ladder operators $B$ and $B^\dagger$ as linear combinations of $A$ and $A^\dagger$ (that is $B = \alpha A + \beta A^\dagger$ with $[B, B^\dagger] = 1$, of course), such that

$$H = \hbar \omega' B^\dagger B + E_0$$

and determine the ground state energy of $E_0$ thereby.

[Hint: You'll need to establish three equations for $|\alpha|$, $|\beta|$, and $\omega'$.]

Problem 4 (30 marks)
Motion along the $x$ axis; position operator $X$, momentum operator $P$. The ground state energy $E_0$ of the Hamilton operator

$$H = \frac{P^2}{2M} + \frac{1}{2} M \omega^2 X^2 + F |X| \quad \text{with } M > 0, \omega > 0, F \text{ arbitrary}$$

is a function of the parameters $M$, $\omega$, and $F$. Determine $\frac{\partial E_0}{\partial F} \big|_{F=0}$. 