Wave motion describes phenomena ranging from the familiar...

Ocean waves...
Sound waves...
Light waves... to the less familiar realm of atomic physics...
...even Quantum Physics (matter waves)...

\[ 3\lambda = 2\pi r \]
... and Relativity (gravity waves).
(1) Making Waves
Water seems to move toward the shore, but no water accumulates on the beach.

What is actually moving?

Do the waves carry energy?
Wave Pulses and Periodic Waves

A Slinky is ideal for studying simple waves.
If a Slinky is laid out on a smooth table with one end held motionless, you can easily produce a single traveling pulse:

- With the Slinky slightly stretched, move the free end back and forth once along the axis of the Slinky.
- You will see a disturbance (the **wave pulse**) move from the free end of the Slinky to the fixed end.

What is actually moving?

- The pulse moves through the Slinky, and portions of the Slinky move as the pulse passes through it.
- After the pulse dies out, the Slinky is exactly where it was before the pulse began.
Moving one end of the Slinky back and forth created a local compression where the rings of the spring are closer together than in the rest of the Slinky.

- This region of compression moves along the Slinky and constitutes the pulse.
- The wave or pulse moves through the medium (here, the Slinky), but the medium goes nowhere.
- What moves is a disturbance within the medium which may be a local compression, a sideways displacement (like a wave on a rope), etc.

The speed of the pulse may depend on factors such as tension in the Slinky and the mass of the Slinky.
Energy is transferred through the Slinky as the pulse travels.

- The work done in moving one end of the Slinky increases both the potential energy of the spring and the kinetic energy of individual loops.
- This region of higher energy then moves along the Slinky and reaches the opposite end.
- There, the energy could be used to ring a bell or perform other types of work.

Energy carried by water waves does substantial work over time in eroding and shaping a shoreline.
The pulse we have been discussing is a **longitudinal wave**: the displacement or disturbance in the medium is parallel to the direction of travel of the wave or pulse.

Sound waves are longitudinal.
By moving your hand up and down, you could also produce a **transverse wave**, in which the displacement or disturbance is **perpendicular** to the direction the wave is traveling.

Waves on a rope and electromagnetic waves are transverse.

Polarization effects are associated with transverse waves but not longitudinal waves.

Water waves have both longitudinal and transverse properties.
If instead of moving your hand back and forth just once, you continue to produce pulses, you will send a series of longitudinal pulses down the Slinky.

- If equal time intervals separate the pulses, you produce a periodic wave.
- The time between pulses is the period $T$ of the wave.
- The number of pulses or cycles per unit of time is the frequency $f = 1/T$.
- The distance between the same points on successive pulses is the wavelength $\lambda$.
- A pulse travels a distance of one wavelength in a time of one period.
- The speed is then the wavelength divided by the period:

$$v = \frac{\lambda}{T} = f \lambda$$
A longitudinal wave traveling on a Slinky has a period of 0.25 s and a wavelength of 30 cm. What is the frequency of the wave?

a) 0.25 Hz  
b) 0.30 Hz  
c) 0.83 Hz  
d) 1.2 Hz  
e) 4 Hz

\[ f = \frac{1}{T} = \frac{1}{0.25 \text{ s}} = 4 \text{ Hz} \]
A longitudinal wave traveling on a Slinky has a period of 0.25 s and a wavelength of 30 cm. What is the speed of the wave?

a) 0.25 cm/s
b) 0.30 cm/s
c) 1 cm/s
d) 7.5 cm/s
e) 120 cm/s

\[ v = f \lambda \]
\[ = (4 \text{ Hz})(30 \text{ cm}) \]
\[ = 120 \text{ cm/s} \]
A snapshot of a single transverse pulse moving along a rope is like a graph of the vertical displacement of the rope plotted against the horizontal position.

- At some later time the pulse will be farther down the rope at a different horizontal position.
- The shape remains basically the same.
If you repeat a series of identical pulses at regular time intervals, you might produce a periodic wave such as shown.

- The wavelength $\lambda$ is the distance covered by one complete cycle of the wave.
- This wave pattern moves to the right along the rope, retaining its shape.
- The shape depends on the exact motion of the hand or other oscillator generating the wave.

When the leading edge of the wave reaches the fixed end of the rope, it will be reflected and start to move back to the left.

- The reflected wave will interfere with the wave still traveling to the right.
If you move your hand up and down smoothly in simple harmonic motion, the displacement of this end of the rope will vary sinusoidally with time.

- The resulting periodic wave will also have a sinusoidal form.
- Such a wave is called a harmonic wave.
- The individual segments of rope tend to move with simple harmonic motion, because the restoring force pulling the rope back toward the center line is proportional to its distance from the center line.

Any periodic wave can be represented as a sum of harmonic waves with different wavelengths and frequencies.

- The process of breaking a complex wave down into its simple harmonic components is called Fourier, or harmonic analysis.
A wave on a rope is shown below. What is the wavelength of this wave?

In 6 m, the wave goes through 2 complete cycles. The wavelength (length of one complete cycle) is \((6 \text{ m})/2 = 3 \text{ m}\).
If the frequency of the wave is 2 Hz, what is the wave speed?

\[ v = f \lambda = (2 \text{ s}^{-1})(3 \text{ m}) = 6 \text{ m/s} \]
As the raised portion of a pulse approaches a given point on the rope, the tension in the rope acquires an upward component.

- The resulting upward force causes this next segment to accelerate upward, and so on down the rope.

The speed of the pulse depends on how fast succeeding segments can be started moving (accelerated).

- By Newton’s second law, this is proportional to the force and inversely proportional to the mass of the segment:
  - A larger tension produces a larger acceleration.
  - The speed of the pulse will increase with the tension and decrease with the mass per unit length of the rope:

\[
\begin{align*}
  a &= \frac{F}{m} \\
  v &= \sqrt{\frac{F}{\mu}} \\
  \text{where } \mu &= \frac{m}{L}
\end{align*}
\]
A rope has an overall length of 10 m and a total mass of 2 kg. The rope is stretched with a tension of 50 N. One end of the rope is fixed, and the other is moved up and down with a frequency of 4 Hz. What is the speed of waves on this rope?

a) 5.0 m/s  b) 7.07 m/s  c) 15.8 m/s  d) 50 m/s  e) 250 m/s

\[ \mu = \frac{m}{L} = \frac{2 \text{ kg}}{10 \text{ m}} = 0.2 \text{ kg/m} \]

\[ v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{50 \text{ N}}{0.2 \text{ kg/m}}} = \sqrt{250 \text{ m}^2/\text{s}^2} = 15.8 \text{ m/s} \]
A rope has an overall length of 10 m and a total mass of 2 kg. The rope is stretched with a tension of 50 N. One end of the rope is fixed, and the other is moved up and down with a frequency of 4 Hz. What is the wavelength?

\[ \nu = f \lambda \implies \lambda = \frac{\nu}{f} = \frac{15.8 \text{ m/s}}{4 \text{ Hz}} = 3.95 \text{ m} \]
When a wave on a rope reaches the fixed end of the rope, it is reflected and travels in the opposite direction back toward your hand.

- If the wave is periodic, the reflected wave interferes with the incoming wave.
- The resulting pattern becomes more complex and confusing.
- This process, in which two or more waves combine, is called interference.
Imagine a rope consisting of two identical segments spliced together to form a single rope of the same mass per unit length as the two original segments.

- Identical waves traveling on the two identical segments will combine to form a larger wave on the single joined rope.
- At all points, the height of the individual waves will add together to form a wave with the same frequency and wavelength but twice the height of the initial two waves.

**Principle of Superposition:**
When two or more waves combine, the resulting disturbance or displacement is equal to the sum of the individual disturbances.
When the two waves are moving the same way at the same time, they are *in phase*.
- The resulting combined wave will be larger (have a greater height).

If one wave is moving upward when the other wave is moving downward, the two waves are *completely out of phase*.
- If the two waves have the same height, the resulting combined displacement will be zero.
- No wave is propagated beyond the junction.

The result of adding two waves together depends on their *phases* as well as on their height or *amplitude*.
- When waves are in phase, we have constructive interference.
- When waves are out of phase, we have destructive interference.
THE PRINCIPLE OF LINEAR SUPERPOSITION

When two or more waves are present simultaneously at the same place, the resultant disturbance is the sum of the disturbances from the individual waves.
Out of phase - destructive Interference

Crest-to-trough

Phase difference ($\Delta \varphi = \pi$)

Resultant Wave

$\frac{\lambda}{2}$

Crest-to-trough
In phase - constructive Interference

Crest-to-crest

Trough-to-trough

\((\Delta \varphi = 2\pi)\)

Resultant Wave
Waves of different wavelengths add like this. Regions of partly constructive and partly destructive interference.
Standing wave modes arise from the combination of reflection and interference such that the reflected waves interfere constructively with the incident waves.
When two or more waves are traveling in the **same direction**, the difference in phase determines whether the interference will be constructive, destructive, or somewhere in between.

When two waves are traveling in **opposite directions**, such as when a wave is reflected back on itself, the principle of superposition can be applied at different points on the string.

- At point A, the two waves cancel each other at all times.
- At this point, the string will not oscillate at all; this is called a **node**.

At point B, both waves will be in phase at all times.

- The two waves always add, producing a displacement twice that of each wave by itself.
- This is called an **antinode**.

\[ \lambda \]

\[ \frac{\lambda}{4} \]

\[ \frac{\lambda}{2} \]

\( \frac{1}{4} \) cycle later
This pattern of oscillation is called a standing wave.

- The waves traveling in opposite directions interfere in a way that produces a standing or fixed pattern.
- The distance between adjacent nodes or adjacent antinodes is half the wavelength of the original waves.
- At the antinodes, the string is oscillating with a large amplitude.
- At the nodes, it is not moving at all.
- At points between the nodes and antinodes, the amplitude has intermediate values.
Review

- **Longitudinal wave vs transverse wave**
- **Harmonic wave**
- **Interference and Standing Waves**
- **Superposition, constructive and destructive interference**
References

- Chapters 16, 17 in Physics